

Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information

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Abstract

This paper studies optimal monetary policy in a model where aggregate fluctuations are driven by the private sector's uncertainty about the economy's fundamentals. Information on aggregate productivity is dispersed across agents and there are two aggregate shocks: a standard productivity shock and a "noise shock" affecting public beliefs about aggregate productivity. Neither the central bank nor individual agents can distinguish the two shocks when they are realized. Despite the lack of superior information, monetary policy can affect the economy's relative response to the two shocks. As time passes, better information on past fundamentals becomes available. The central bank can then adopt a backward-looking policy rule, based on more precise information about past shocks. By announcing its response to future information, the central bank can influence the expected real interest rate faced by forward-looking consumers with different beliefs and thus affect the equilibrium allocation. If the announced future response is sufficiently aggressive, the central bank can completely eliminate the effect of noise shocks. However, this policy is typically suboptimal, as it leads to an excessively compressed distribution of relative prices. The optimal monetary policy balances the benefits of aggregate stabilization with the costs in terms of cross-sectional efficiency.

Keywords: Monetary policy, imperfect information, consumer sentiment.

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1 Introduction

Suppose a central bank observes an unexpected expansion in economic activity. This could be due to a shift in fundamentals, say an aggregate productivity shock, or to a shift in public beliefs with no actual change in the economy's fundamentals. If the central bank could tell apart the two shocks the optimal response would be simple: accommodate the first shock and offset the second. In reality, however, central banks can rarely tell apart these shocks when they hit the economy. What can the central bank do in this case? What is the optimal monetary policy response? In this paper, I address these questions in the context of a model with dispersed information, which allows for a micro-founded treatment of fundamental and "sentiment" shocks.

The US experience in the second half of the 90s has fueled a lively debate on these issues. The run up in asset prices has been taken by many as a sign of optimistic expectations about widespread technological innovations. In this context, the advice given by different economists has been strongly influenced by the assumptions made on the ability of the central bank to identify the economy's actual fundamentals. Some, e.g., Cecchetti et al. (2000) and Dupor (2005), attribute to the central bank some form of superior information and advocate early intervention to contain an expansion driven by incorrect beliefs. Others, e.g., Bernanke and Gertler (2001), emphasize the uncertainty associated with the central bank's decisions and advocate sticking to a simple inflation targeting rule. In this paper, I explore the idea that, even if the central bank does not have superior information, a policy rule can be designed to take into account, and partially offset, aggregate mistakes by the private sector regarding the economy's fundamentals.

I consider an economy with heterogeneous agents and monopolistic competition, where aggregate productivity is subject to unobservable shocks. Agents have access to a noisy public signal of aggregate productivity, which summarizes public news about technological advances, aggregate statistics, and information reflected in stock market prices and other financial variables. The error term in this signal introduces aggregate "noise shocks," that is, shocks to public beliefs which are uncorrelated with actual productivity shocks. In addition to the public signal, agents have access to private information regarding the realized productivity in the sector where they work. Due to cross-sectional heterogeneity, this information is not sufficient to identify the value of the aggregate shock. Therefore, agents combine public and private sources of information to forecast the aggregate behavior of the economy. The central bank

has only access to public information.

In this environment, I obtain two sets of results. First, I show that the monetary authority, using a policy rule which responds to past aggregate shocks, has the power to change the relative response of the economy to productivity and noise shocks. Actually, there exists a policy rule that perfectly replicates the full information level of aggregate activity. I dub this policy “full aggregate stabilization.” Second, I derive the optimal policy rule and show that full aggregate stabilization is typically suboptimal. In particular, if the coefficient of relative risk aversion is greater or equal than one, at the optimal policy, aggregate output responds less than proportionally to changes in aggregate fundamentals and responds positively to noise shocks.

The fact that monetary policy can tackle the two shocks separately is due to two crucial ingredients. First, agents are forward looking. Second, productivity shocks are unobservable when they are realized, but become public knowledge in later periods. At that point, the central bank can respond to them. By choosing an appropriate policy rule the monetary authority can then alter the way in which agents respond to private and public information. Specifically, the monetary authority can announce that it will increase its price level target following an actual increase in aggregate productivity today. Under this policy, consumers observing an increase in productivity in their own sector expect higher inflation than consumers who only observe a positive public signal. Therefore, they expect a lower real interest rate and choose to consume more. This makes consumption more responsive to private information and less to public information and moderates the economy’s response to noise shocks. This result points to an idea which applies more generally in models with dispersed information. If future policy is set contingent on variables that are imperfectly observed today, this can change the agents’ reaction to different sources of information, and thus affect the equilibrium allocation.

In the model presented, the power of policy rules to shape the economy’s response to aggregate shocks is surprisingly strong. Namely, by adopting the appropriate rule the central bank can support an equilibrium where aggregate output responds one for one to fundamentals and does not respond at all to noise in public news. However, such a policy is typically suboptimal for its undesirable consequences in terms of the cross-sectional allocation. In particular, full aggregate stabilization generates an inefficient compression in the distribution of relative prices.

The equilibrium under the optimal monetary policy achieves a constrained efficient allocation. To define the appropriate benchmark for constrained efficiency, I consider a social

planner who can dictate the way in which individual consumers respond to the information in their hands, but cannot change their access to information, as in Hellwig (2005) and Angeletos and Pavan (2007). I then show that, in a general equilibrium environment with isoelastic preferences and Gaussian shocks, a simple linear monetary policy rule, together with a non-state-contingent production subsidy, are enough to eliminate all distortions due to dispersed information and monopolistic competition. In particular, a policy rule that only depends on aggregate variables is enough to induce agents to make an optimal use of public and private information.¹

Finally, I use the model to ask whether better public information can have destabilizing effects on the economy and whether it can lead to social welfare losses. This connects the paper to the growing debate on the social value of public information, started by Morris and Shin (2002).² I show that increasing the precision of the public signal increases the response of aggregate output to noise shocks and can potentially increase output gap volatility (where the gap is measured as the distance from the full information equilibrium). However, as agents receive more precise information on average productivity, they also set relative prices that are more responsive to individual productivity differences. Therefore, a more precise public signal improves welfare by allowing a more efficient cross-sectional distribution of consumption and labor effort. What is the total welfare effect of increasing the public signal's precision? If monetary policy is kept constant, then a more precise public signal can, for some set of parameters, reduce total welfare. This provides an interesting general equilibrium counterpart to Morris and Shin's (2002) "anti-transparency" result. However, if monetary policy is chosen optimally, then a more precise signal is always welfare improving. This follows the general principle, pointed out in Angeletos and Pavan (2007), that more precise information is always desirable when the equilibrium is constrained efficient.

A number of recent papers, starting with Woodford (2002) and Sims (2003), have revived the study of monetary models with imperfect common knowledge, in the tradition of Phelps (1969) and Lucas (1972).³ In particular, this paper is more closely related to Hellwig (2005)

¹Angeletos and Pavan (2009) derive a similar result in the context of quadratic games. See Angeletos, Lorenzoni, and Pavan (2008) for an application of the same principle to a model of investment and financial markets.

²See Angeletos and Pavan (2007, 2009), Amador and Weill (2007), Hellwig and Veldkamp (2009). For applications to the transparency of monetary policy, see Amato, Morris, and Shin (2002), Svensson (2005), Hellwig (2005), Morris and Shin (2005).

³See also Moscarini (2004), Milani (2007), Nimark (2007), Bacchetta and Van Wincoop (2008), Luo (2008), Maćkowiak and Wiederholt (2009). Mankiw and Reis (2002) and Reis (2006) explore the complementary idea of lags in informational adjustment as a source of nominal rigidity.

and Adam (2007), who study monetary policy in economies where money supply is imperfectly observed by the public. In both papers consumers' decisions are essentially static, as a cash-in-advance constraint is present and always binding. Therefore, the forward-looking element which is crucial in this paper, is absent in their models. In the earlier literature, King (1982) was the first to recognize the power of policy rules in models with imperfect information. He noticed that “*prospective* feedback actions” responding to “disturbances that are currently imperfectly known by agents” can affect real outcomes (King, 1982, p. 248). The mechanism in King (1982) is based on the fact that different policy rules change the informational content of prices. As I will show below, that channel is absent in this paper. Here, policy rules matter only because they affect agents' incentives to respond to private and public signals. Angeletos and La'O (2008) explore a general equilibrium model related to the one in this paper, but where the precision of the information revealed by prices is endogenous, and focus on the distortions generated by this endogeneity.

The existing literature on optimal monetary policy with uncertain fundamentals has focused on the case of common information in the private sector (Aoki, 2003, Orphanides, 2003, Svensson and Woodford, 2003, 2004, and Reis, 2003). A distinctive feature of the model in this paper is that private agents have access to superior information about fundamentals in their local market but not in the aggregate. The presence of dispersed information generates a novel tension between aggregate efficiency and cross-sectional efficiency in the design of optimal policy.

There is a growing literature on expectation driven business cycles (e.g., Beaudry and Portier, 2006, and Jaimovich and Rebelo, 2006). In particular, Lorenzoni (2009) shows that noise shocks affecting the private sector's expectations about aggregate productivity can generate realistic aggregate demand disturbances in a business cycle model with nominal rigidities. However, the role of these noise shocks depend on the monetary policy response. This leads to the question: are noise driven cycles merely a symptom of a suboptimal monetary regime or do they survive under optimal monetary policy? The welfare analysis in this paper shows that optimal policy does not eliminate the effect of noise shocks.⁴

Finally, from a methodological point of view, this paper is related to a set of papers who exploit isoelastic preferences and Gaussian shocks to derive closed-form expressions for social

⁴Imperfect information is an important ingredient for this argument. Christiano, Motto, and Rostagno (2006) analyze a full information model where business cycles are driven by news about future productivity. In Appendix B of their paper, they show that optimal monetary policy essentially mutes the effects of those news shocks.

welfare in heterogeneous agent economies, e.g., Benabou (2002) and Heathcote, Storesletten, and Violante (2008). The main novelty here is the presence of differentiated goods and consumer-specific consumption baskets.

The model is introduced in Section 2. In Section 3, I characterize stationary, linear rational expectations equilibria. In Section 4, I show how the choice of the monetary policy rule affects the equilibrium allocation. In Section 5, I derive the welfare implications of different policies, characterize optimal monetary policy, and prove constrained efficiency. In Section 6, I study the welfare effects of public information. Section 7 concludes. All the proofs not in the text are in the appendix.

2 The Model

2.1 Setup

I consider a dynamic model of monopolistic competition à la Dixit-Stiglitz with heterogeneous productivity shocks and imperfect information regarding aggregate shocks. Prices are set at the beginning of each period, but are otherwise flexible.

There is a continuum of infinitely lived households uniformly distributed on $[0, 1]$. Each household i is made of two agents: a consumer and a producer specialized in the production of good i . Preferences are represented by the utility function

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t U(C_{it}, N_{it}) \right],$$

with

$$U(C_{it}, N_{it}) = \frac{1}{1-\gamma} C_{it}^{1-\gamma} - \frac{1}{1+\eta} N_{it}^{1+\eta},$$

where C_{it} is a consumption index and N_{it} is the labor effort of producer i .⁵ The consumption index is given by

$$C_{it} = \left(\int_{J_{it}} C_{ijt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}},$$

where C_{ijt} denotes consumption of good j by consumer i in period t , and $J_{it} \subset [0, 1]$ is a random consumption basket, described in detail below. The elasticity of substitution between goods, σ , is greater than 1.

The production function for good i is

$$Y_{it} = A_{it} N_{it}.$$

⁵As usual, when $\gamma = 1$ the per-period utility function is $U(C_{it}, N_{it}) = \log C_{it} - (1/(1+\eta))N_{it}^{1+\eta}$.

Productivity is household-specific and labor is immobile across households. The productivity parameters A_{it} are the fundamental source of uncertainty in the model. Let a_{it} denote the log of individual productivity, $a_{it} = \log A_{it}$. Throughout the paper, a lowercase variable will denote the natural logarithm of the corresponding uppercase variable. Individual productivity has an aggregate component a_t and an idiosyncratic component ϵ_{it} ,

$$a_{it} = a_t + \epsilon_{it},$$

with $\int_0^1 \epsilon_{it} di = 0$. Aggregate productivity a_t follows the AR1 process

$$a_t = \rho a_{t-1} + \theta_t,$$

with $\rho \in [0, 1]$.

At the beginning of period t , all households observe the value of aggregate productivity in the previous period, a_{t-1} . Next, the shocks θ_t and ϵ_{it} are realized. Agents in household i do not observe θ_t and ϵ_{it} separately, they only observe the sum of the two, that is, the individual productivity innovation

$$x_{it} = \theta_t + \epsilon_{it}.$$

Moreover, all agents observe a noisy public signal of the aggregate innovation

$$s_t = \theta_t + e_t.$$

The random variables θ_t , ϵ_{it} and e_t are independent, serially uncorrelated, and normally distributed with zero mean and variances σ_θ^2 , σ_ϵ^2 , and σ_e^2 .⁶ I assume throughout the paper that σ_θ^2 and σ_e^2 are strictly positive, and I study separately the cases $\sigma_e^2 = 0$ and $\sigma_e^2 > 0$, corresponding, respectively, to full information and imperfect information on θ_t .

Summarizing, there are two aggregate shocks: the productivity shock θ_t and the noise shock e_t . Both are unobservable during period t and are fully revealed at the beginning of $t + 1$, when a_t is observed. The second shock is a source of correlated mistakes, as it induces households to temporarily overstate or understate the current value of θ_t .

In the appendix, I give a full description of the matching process between consumers and producers. Here, I summarize the properties of the consumption baskets that arise from the

⁶In the cases where $\gamma \neq 1$ and productivity is a random walk, $\rho = 1$, it is necessary to impose a bound on σ_θ^2 to ensure that expected utility is finite, namely

$$\sigma_\theta^2 < 2 \left(\frac{\gamma + \eta}{(1 - \gamma)(1 + \eta)} \right)^2 (-\log \beta).$$

process. Each period, each consumer i is assigned an unobservable sampling shock v_{it} . Then, nature selects a random subset of goods $J_{it} \subset [0, 1]$ of fixed measure, with the following property: the distribution of productivity shocks ϵ_{jt} for the goods in J_{it} is normal with mean v_{it} and variance $\sigma_{\epsilon|v}^2$. The sampling shocks v_{it} are normally distributed across consumers, with zero mean and variance σ_v^2 . They are independent of all other shocks and satisfy $\int_0^1 v_{it} di = 0$. To ensure consistency of the matching process, the variances $\sigma_v^2, \sigma_{\epsilon|v}^2$ and σ_ϵ^2 have to satisfy $\sigma_v^2 + \sigma_{\epsilon|v}^2 = \sigma_\epsilon^2$. Therefore, the variance σ_v^2 is restricted to be in the interval $[0, \sigma_\epsilon^2]$. The parameter $\chi = \sigma_v^2 / \sigma_\epsilon^2 \in [0, 1]$ reflects the degree of heterogeneity in consumption baskets. The limit cases $\chi = 0$ and $\chi = 1$ correspond, respectively, to the case where every consumer consumes a representative sample of the goods in the economy and to the case where every consumer consumes a sample of goods with identical productivity.

2.2 Trading, financial markets and monetary policy

The central bank acts as an account keeper for the agents in the economy. Each household holds an account denominated in dollars, directly with the central bank. The account is debited whenever the consumer makes a purchase and credited whenever the producer makes a sale. The balance of household i at the beginning of the period is denoted by B_{it} . All households begin with a zero balance at date 0. At the beginning of each period t , the bank sets the (gross) nominal interest rate R_t , which will apply to end-of-period balances. Households are allowed to hold negative balances at the end of the period and the same interest rate applies to positive and negative balances. However, there is a lower bound on nominal balances, which rules out Ponzi schemes.

To describe the trading environment, it is convenient to divide each period in three stages, $(t, 0)$, (t, I) , and (t, II) . In stage $(t, 0)$, everybody observes a_{t-1} , the central bank sets R_t , and households trade one-period state-contingent claims on a centralized financial market. These claims will be paid in $(t+1, 0)$. In stages (t, I) and (t, II) , the market for state-contingent claims is closed and the only trades allowed are trades of goods for nominal balances. In (t, I) , all aggregate and individual shocks are realized, producer i observes s_t and x_{it} , sets the dollar price of good i , P_{it} , and stands ready to deliver any quantity of good i at that price. In (t, II) , consumer i observes the prices of the goods in his consumption basket, $\{P_{jt}\}_{j \in J_{it}}$, chooses his consumption vector, $\{C_{ijt}\}_{j \in J_{it}}$, and buys C_{ijt} from each producer $j \in J_{it}$. In this stage, consumer i and producer i are spatially separated, so the consumer does not observe the current production of good i . Figure 1 summarizes the events taking place during period t .

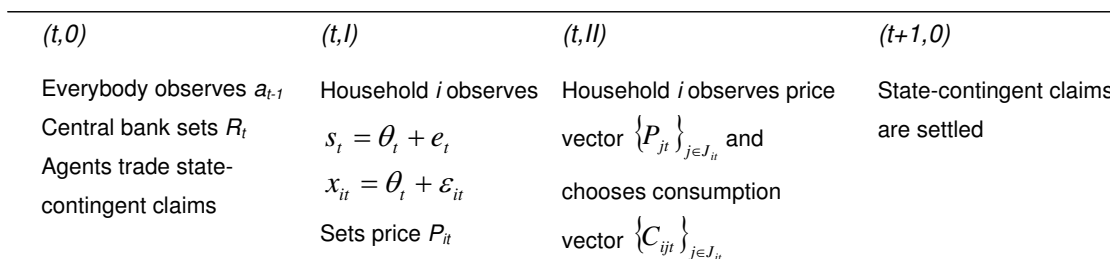


Figure 1: Timeline

Let $Z_{it+1}(\omega_{it})$ denote the state-contingent claims purchased by household i in $(t, 0)$, where $\omega_{it} \equiv (\epsilon_{it}, v_{it}, \theta_t, e_t)$. The price of these claims is denoted by $Q_t(\omega_{it})$. The household balances at the beginning of period $t + 1$ are then given by

$$B_{it+1} = R_t \left[B_{it} - \int_{\mathbb{R}^4} Q_t(\tilde{\omega}_{it}) Z_{it+1}(\tilde{\omega}_{it}) d\tilde{\omega}_t + (1 + \tau) P_{it} Y_{it} - \int_{J_{it}} P_{jt} C_{ijt} dj - T_t \right] + Z_{it+1}(\omega_{it}),$$

where τ is a proportional subsidy on sales and T_t is a lump-sum tax.

Since households are exposed to idiosyncratic risk, they will generally end up with different end-of-period balances. However, since they face identical shocks ex ante, they can fully insure by trading contingent claims in $(t, 0)$. Therefore, beginning-of-period balances will be constant and equal to 0 in equilibrium. This eliminates the wealth distribution from the state variables of the problem, which greatly simplifies the analysis.⁷

Let me define aggregate indexes for nominal prices and real activity. For analytical conve-

⁷The use of this type of assumption to simplify the study of monetary models goes back to Lucas (1990).

nience, I use simple geometric means,⁸

$$P_t \equiv \exp \left(\int_0^1 \log P_{it} di \right),$$

$$C_t \equiv \exp \left(\int_0^1 \log C_{it} di \right).$$

The behavior of the monetary authority is described by a policy rule. In period $(t, 0)$, the central bank sets R_t based on the past realizations of the exogenous shocks θ_t and e_t , and on the past realizations of P_t and C_t . The monetary policy rule is described by the map $R_t = \mathcal{R}(h_t, P_{t-1}, C_{t-1}, \dots, P_0, C_0)$, where h_t denotes the vector of past aggregate shocks

$$h_t \equiv (\theta_{t-1}, e_{t-1}, \theta_{t-2}, e_{t-2}, \dots, \theta_0, e_0).$$

Allowing the central bank to condition R_t on the current public signal s_t would not alter any of the results. The only other policy instrument available is the subsidy τ , which is financed by the lump-sum tax T_t . The government runs a balanced budget so

$$T_t = \tau \int_0^1 P_{it} Y_{it} di.$$

As usual in the literature, the subsidy τ will be used to eliminate the distortions due to monopolistic competition.

2.3 Equilibrium definition

Household behavior is captured by three functions, \mathcal{Z} , \mathcal{P} and \mathcal{C} . The first gives the optimal holdings of state-contingent claims as a function of the initial balances B_{it} and of the vector of past aggregate shocks h_t , that is, $Z_{it+1}(\omega_{it}) = \mathcal{Z}(\omega_{it}; B_{it}, h_t)$. The second gives the optimal price for household i , as a function of the same variables plus the current realization of individual productivity and of the public signal, $P_{it} = \mathcal{P}(B_{it}, h_t, s_t, x_{it})$. The third gives optimal consumption as a function of the same variables plus the observed price vector, $C_{it} = \mathcal{C}(B_{it}, h_t, s_t, x_{it}, \{P_{ij}\}_{j \in J_{it}})$. Before defining an equilibrium, I need to introduce two other objects. Let $\mathcal{D}(\cdot|h_t)$ denote the distribution of nominal balances B_{it} across households,

⁸Alternative price and quantity indexes are

$$P_t^\circ \equiv \left(\int_0^1 P_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}},$$

$$Y_t^\circ \equiv \frac{\int_0^1 P_{it} Y_{it} di}{P_t^\circ}.$$

All results stated for P_t and C_t hold for P_t° and Y_t° , modulo multiplicative constants.

conditional on the history of past aggregate shocks h_t . The price of a ω_{it} -contingent claim in period $(t, 0)$, given the vector of past shocks h_t , is given by $\mathcal{Q}(\omega_{it}; h_t)$.

A *symmetric rational expectations equilibrium* under the policy rule \mathcal{R} is given by an array $\{\mathcal{Z}, \mathcal{P}, \mathcal{C}, \mathcal{D}, \mathcal{Q}\}$ that satisfies three conditions: optimality, market clearing, and consistency. Optimality requires that the individual rules \mathcal{Z} , \mathcal{P} , and \mathcal{C} are optimal for the individual household, taking as given: the exogenous law of motion for h_t , the policy rule \mathcal{R} , the prices \mathcal{Q} , and the fact that all other households follow \mathcal{Z} , \mathcal{P} , and \mathcal{C} , and that their nominal balances are distributed according to \mathcal{D} . Market clearing requires that the goods markets and the market for state-contingent claims clear for each h_t . Consistency requires that the dynamics of the distribution of nominal balances, described by \mathcal{D} , are consistent with the individual decision rules.

3 Linear equilibria

In this section, I characterize the equilibrium behavior of output and prices. Given the assumption of complete financial markets in $(t, 0)$, I can focus on equilibria where beginning-of-period nominal balances are constant and equal to zero for all households. That is, the distribution $\mathcal{D}(\cdot|h_t)$ is degenerate for all h_t . Moreover, thanks to the assumption of separable, isoelastic preferences and Gaussian shocks, it is possible to derive linear rational expectations equilibria in closed form. In particular, I will consider equilibria where individual prices and consumption levels are, in logs,

$$p_{it} = \phi_a a_{t-1} + \phi_s s_t + \phi_x x_{it}, \quad (1)$$

$$c_{it} = \psi_0 + \psi_a a_{t-1} + \psi_s s_t + \psi_x x_{it} + \psi_{\bar{x}} \bar{x}_{it}, \quad (2)$$

where $\phi \equiv \{\phi_a, \phi_s, \phi_x\}$ and $\psi \equiv \{\psi_0, \psi_a, \psi_s, \psi_x, \psi_{\bar{x}}\}$ are vectors of constant coefficients to be determined and \bar{x}_{it} is the average productivity innovation for the goods in the basket of consumer i ,

$$\bar{x}_{it} \equiv \int_{J_{it}} x_{jt} dj = \theta_t + v_{it}.$$

I will explain in a moment why this variable enters (2). Summing (1) and (2) across agents, gives the aggregate price and quantity indexes

$$p_t = \phi_a a_{t-1} + \phi_{\theta} \theta_t + \phi_s e_t, \quad (3)$$

$$c_t = \psi_0 + \psi_a a_{t-1} + \psi_{\theta} \theta_t + \psi_s e_t, \quad (4)$$

where $\phi_{\theta} \equiv \phi_s + \phi_x$ and $\psi_{\theta} \equiv \psi_s + \psi_x + \psi_{\bar{x}}$.

3.1 Optimal prices and consumption

Consumer i faces the vector of log prices $\{p_{jt}\}_{j \in J_{it}}$. If all producers follow the linear rule (1), these prices are normally distributed with mean $\phi_a a_{t-1} + \phi_s s_t + \phi_x \bar{x}_{it}$ and variance $\phi_x^2 \sigma_{\epsilon|v}^2$. This follows from the assumption on consumption baskets and has two useful implications. First, it is possible to derive an exact expression for the price index of consumer i which, in logs, takes the form

$$\bar{p}_{it} = \kappa_{\bar{p}} + p_t + \phi_x v_{it}, \quad (5)$$

where $\kappa_{\bar{p}}$ is a constant term derived explicitly in Lemma 5, in the appendix. Second, as the consumer already knows a_{t-1} and s_t , he can back out $\phi_x \bar{x}_{it}$ from the mean of this distribution and this is a sufficient statistic for all the information on θ_t contained in the observed prices. This proves the following lemma.

Lemma 1 *If prices are given by (1), then the information of consumer i regarding the current shock θ_t is summarized by the three independent signals s_t, x_{it} and $\phi_x \bar{x}_{it}$.*

In a linear equilibrium it is possible to write the household's first-order conditions for P_{it} and C_{it} in a linear form. Detailed derivations and explicit expressions for the constant terms κ_p and κ_c below, are in the proof of Proposition 1, in the appendix. Optimal price-setting gives

$$p_{it} = \kappa_p + \mathbb{E}_{i,(t,I)} [\bar{p}_{it} + \gamma c_{it} + \eta n_{it}] - a_{it}, \quad (6)$$

where $\mathbb{E}_{i,(t,I)} [\cdot]$ denotes the expectation of producer i at date (t, I) . The expression on the right-hand side of (6) captures the expected nominal marginal cost plus a constant mark-up. The nominal marginal cost depends positively on the price index \bar{p}_{it} and on the marginal rate of substitution between consumption and leisure $\gamma c_{it} + \eta n_{it}$, and negatively on productivity a_{it} . All the relevant information needed to compute the expectation in (6) is summarized by a_{t-1}, s_t and x_{it} , so $\mathbb{E}_{i,(t,I)} [\cdot]$ can be replaced by $\mathbb{E} [\cdot | a_{t-1}, s_t, x_{it}]$.

The optimality condition for C_{it} takes the form

$$c_{it} = \kappa_c + \mathbb{E}_{i,(t,II)} [c_{it+1}] - \gamma^{-1} (r_t - \mathbb{E}_{i,(t,II)} [\bar{p}_{it+1}] + \bar{p}_{it}), \quad (7)$$

where $\mathbb{E}_{i,(t,II)} [\cdot]$ denotes the expectation of consumer i at date (t, II) . Apart from the fact that expectations and price indexes are consumer-specific, this is a standard Euler equation: current consumption depends positively on future expected consumption and negatively on the expected real interest rate. Lemma 1 implies that $\mathbb{E}_{i,(t,II)} [\cdot]$ can be replaced

by $\mathbb{E}[\cdot|a_{t-1}, s_t, x_{it}, \phi_x \bar{x}_{it}]$, confirming the initial conjecture that individual consumption is a linear function of a_{t-1} , s_t , x_{it} , and \bar{x}_{it} .

3.2 Policy rule and equilibrium

To find an equilibrium, I substitute (1) and (2) in the optimality conditions (6) and (7), and obtain a system of equations in ϕ and ψ .⁹ This system does not determine ϕ and ψ uniquely. In particular, for any choice of the parameter ϕ_a in \mathbb{R} , there is a unique pair $\{\phi, \psi\}$ which is consistent with individual optimality. To complete the equilibrium characterization and pin down ϕ_a , I need to specify the monetary policy rule.

Consider an interest rate rule which targets the aggregate price level. The nominal interest rate is set to

$$r_t = \xi_0 + \xi_a a_{t-1} + \xi_p (p_{t-1} - \hat{p}_{t-1}), \quad (8)$$

where \hat{p}_t is the central bank's target

$$\hat{p}_t = \mu_a a_{t-1} + \mu_\theta \theta_t + \mu_e e_t. \quad (9)$$

The parameters $\{\xi_0, \xi_a, \xi_p\}$ and $\{\mu_a, \mu_\theta, \mu_e\}$ are chosen by the monetary authority. The central bank's behavior can be described as follows. At the beginning of period t , the monetary authority observes a_{t-1} and announces its current target \hat{p}_t for the price level. The target \hat{p}_t has a forecastable, backward-looking component $\mu_a a_{t-1}$, and a state-contingent part which is allowed to respond to the current shocks θ_t and e_t . During trading, each agent sets his price and consumption responding to the variables in his information set. At the beginning of period $t+1$, the central bank observes the realized price level p_t and the realized shocks θ_t and e_t . If p_t deviated from target in period t , the next period nominal interest rate is adjusted according to (8).

Given this policy rule, I can complete the equilibrium characterization and prove the existence of stationary linear equilibria. In particular, the next proposition shows that the choice of μ_a by the monetary authority pins down ϕ_a and thus the equilibrium coefficients ϕ and ψ . The choice of μ_a also pins down the remaining coefficients in the policy rule, except ξ_p . The choice of this parameter does not affect the equilibrium allocation, it only affects the local determinacy properties of the equilibrium. Notice that the proposition excludes one possible value for μ_a , denoted by μ_a^0 , corresponding to the pathological case where the equilibrium construction would give $\phi_x = 0$. This case is discussed in the appendix.

⁹See (31)-(38) in the appendix.

Proposition 1 *For each $\mu_a \in \mathbb{R} / \{\mu_a^0\}$ there exist a pair $\{\phi, \psi\}$ and a vector $\{\xi_0, \xi_a, \mu_\theta, \mu_e\}$ such that the prices and consumption levels (1)-(2) form a rational expectations equilibrium under the policy rule (8)-(9), for any $\xi_p \in \mathbb{R}$. If $\xi_p > 1$ the equilibrium is locally determinate. The value of ψ_a is independent of the policy rule and equal to*

$$\psi_a = \frac{1 + \eta}{\gamma + \eta} \rho.$$

In equilibrium, $p_t = \hat{p}_t$ and the interest rate is equal to

$$r_t = \xi_0 - (\mu_a + \gamma\psi_a)(1 - \rho) a_{t-1}.$$

4 The effects of monetary policy

Let me turn now to the effects of different policy rules on the equilibrium allocation. By Proposition 1, the choice of the policy rule is summarized by the parameter μ_a , so, from now on, I will simply refer to the policy rule μ_a . Proposition 1 shows that in equilibrium the central bank always achieves its price level target. Therefore, by choosing μ_a the central bank determines the aggregate response of prices to past realizations of aggregate productivity. Since a_{t-1} is common knowledge at time t , price setters can easily coordinate on setting prices proportional to $\exp\{\mu_a a_{t-1}\}$.

The first question raised in the introduction can now be stated in formal terms. How does the choice of μ_a affect the equilibrium response of aggregate activity to fundamental and noise shocks, that is, the coefficients ψ_θ and ψ_s in (4)? More generally, how does the choice of μ_a affect the vectors ϕ and ψ , which determine the cross-sectional allocation of goods and labor effort across households? The rest of this section addresses these questions.

4.1 Full information

Let me begin with the case where households have full information on θ_t . This happens when s_t is a noiseless signal, $\sigma_e^2 = 0$. In this case, households can perfectly forecast current aggregate prices and consumption, p_t and c_t , by observing a_{t-1} and s_t . Taking the expectation $\mathbb{E}[\cdot | a_{t-1}, s_t]$ on both sides of the optimal pricing condition (6) and omitting the constant terms, gives

$$p_t = p_t + \gamma c_t + \eta(c_t - a_t) - a_t.$$

This implies that aggregate consumption under full information, denoted by c_t^{fi} , must satisfy

$$c_t^{fi} = \psi_0 + \frac{1 + \eta}{\gamma + \eta} a_t, \tag{10}$$

that is, $\psi_a = \rho(1 + \eta) / (\gamma + \eta)$ and $\psi_\theta = (1 + \eta) / (\gamma + \eta)$. In the next proposition, I show that the other coefficients affecting the equilibrium allocation are also uniquely determined and independent of μ_a . This is a baseline neutrality result: under full information the equilibrium allocation of consumption goods and labor effort is independent of the monetary policy rule.¹⁰

Proposition 2 *When the signal s_t is perfectly informative, $\sigma_e^2 = 0$, the equilibrium allocation is independent of the monetary policy rule μ_a .*

4.2 Imperfect information

Let me turn now to the case of imperfect information, which arises when σ_e^2 is positive. In this case, the choice of μ_a does affect the equilibrium allocation. To understand how monetary policy operates, it is useful to start from a special case.

Consider the case where the intertemporal elasticity of substitution is $\gamma = 1$, the disutility of effort is linear, $\eta = 0$, productivity is a random walk, $\rho = 1$, and there is no heterogeneity in consumption baskets, $\chi = 0$.¹¹ In this case, the nominal interest rate is constant and the Euler equation (7) becomes

$$c_{it} = \mathbb{E}_{i,(t,II)} [c_{it+1}] + \mathbb{E}_{i,(t,II)} [p_{t+1}] - p_t.$$

Given that future shocks have zero expected value at time t and $\psi_a = 1$, from Proposition 1, equation (2) implies that the expected future consumption on the right-hand side is equal to $\mathbb{E}_{i,(t,II)} [\psi_0 + a_t]$. Moreover, under the price level target (9), the expected future price level is equal to $\mu_a \mathbb{E}_{i,(t,II)} [a_t]$. As I will show below, with homogeneous consumption baskets, consumers can perfectly infer the value of θ_t from the observed values of p_t and s_t , so $\mathbb{E}_{i,(t,II)} [a_t] = a_t$. Putting these results together, it follows that all consumers choose the same consumption level

$$c_t = \psi_0 + (1 + \mu_a) a_t - p_t. \tag{11}$$

At the price-setting stage, households still have imperfect information, given that they only observe s_t and x_{it} . Substituting for consumption, using (11), the optimal pricing condition (6) becomes

$$p_{it} = (1 + \mu_a) \mathbb{E} [a_t | s_t, x_{it}] - a_{it}.$$

¹⁰See McCallum (1979) for an early neutrality result in a model with pre-set prices.

¹¹The following shows that the basic positive result of the paper can be derived with homogeneous consumption baskets. However, Proposition 6 below shows that heterogeneous consumption baskets are necessary to obtain interesting welfare trade-offs.

The expectation on the right-hand side can be written as $a_{t-1} + \beta_s s_t + \beta_x x_{it}$, where β_s and β_x are positive coefficients that satisfy $\beta_s + \beta_x < 1$.¹² Aggregating across producers and rearranging gives

$$p_t = \mu_a a_{t-1} + (1 + \mu_a) \beta_s s_t + ((1 + \mu_a) \beta_x - 1) \theta_t. \quad (12)$$

This shows that p_t and s_t fully reveal θ_t , except in the knife-edge case where $\mu_a = 1/\beta_x - 1$, which I will disregard.

Finally, combining (11) and (12) gives an expression for equilibrium consumption in terms of exogenous shocks

$$c_t = \psi_0 + a_{t-1} + (1 + (1 + \mu_a)(1 - \beta_s - \beta_x)) \theta_t - (1 + \mu_a) \beta_s e_t. \quad (13)$$

Therefore, the responses of aggregate consumption to fundamental and noise shock are $\psi_\theta = 1 + (1 + \mu_a)(1 - \beta_s - \beta_x)$ and $\psi_s = -(1 + \mu_a) \beta_s$ and the choice of the policy rule μ_a is no longer neutral. In particular, increasing μ_a increases the output response to fundamental shocks and reduces the response to noise shocks.

To interpret this result, it is useful to look separately at consumers' and price setters' behavior. If the monetary authority increases μ_a , equation (11) shows that, for a given price level p_t , the response of consumer spending to θ_t increases. A larger value of μ_a implies that, if a positive productivity shock materializes at date t , the central bank will target a higher price level in the following period. This, translates in a lower expected real interest rate, leading to higher current spending. On the other hand, the consumers' response to a noise shock e_t , for given p_t , is zero irrespective of μ_a , given that consumers have perfect information on a_t and place zero weight on the signal s_t .

Consider now the response of price setters. If the monetary authority chooses a larger value for μ_a , price setters tend to set higher prices following a positive productivity shock θ_t as they observe a positive s_t and, on average, a positive x_{it} , and thus expect higher consumer spending. However, due to imperfect information, they tend to underestimate the spending increase. Therefore, their price increase is not enough to undo the direct effect on consumers' demand, and, on net, real consumption goes up. Formally, this is captured by

$$\frac{\partial \psi_\theta}{\partial \mu_a} = 1 - \beta_s - \beta_x > 0.$$

On the other hand, following a positive noise shocks, price setters mistakenly expect an increase in demand, following their observation of a positive s_t , and tend to raise prices. Consumers'

¹²See (29) in the appendix.

demand, however, is unchanged. The net effect is a reduction in output, that is,

$$\frac{\partial \psi_s}{\partial \mu_a} = -\beta_s < 0.$$

These results extend to the general case, as proved in the following proposition.

Proposition 3 *When the signal s_t is noisy, $\sigma_e^2 > 0$, the equilibrium allocation depends on μ_a . The coefficients $\{\phi, \psi\}$ are linear functions of the policy parameter μ_a , with*

$$\frac{\partial \psi_\theta}{\partial \mu_a} > 0, \quad \frac{\partial \psi_s}{\partial \mu_a} < 0, \quad \frac{\partial \phi_x}{\partial \mu_a} > 0, \quad \frac{\partial \phi_s}{\partial \mu_a} > 0.$$

The intuition for the special case extends to the general case. In particular, it is not necessary for the result that consumers have perfect information on θ_t . What is crucial is that price setters find it easier to forecast a demand increase driven by the public signal s_t , relative to a demand increase driven by the consumers' private signals x_{it} and \bar{x}_{it} . When μ_a is larger, consumers expect an increase in nominal prices at $t + 1$ following any positive signal about future productivity, either public or private. If a positive fundamental shock hits, consumers' expectations are driven both by public and private signals. The producers can perfectly forecast the demand increase associated to the public signal, but can only partially foresee the demand increase due to private signals. Therefore, average current prices increase less than expected future prices, the average expected real interest rate drops and real output increases. If, instead, a positive noise shock hits, consumers' expectations on future prices are only driven, on average, by the public signal. The producers, observing the public signal, adjust upwards their expectation of θ_t and forecast a demand increase driven by both public and private signals. Therefore, current prices tend to increase more than expected future prices, the average expected real interest rate increases and real output falls.

Three crucial ingredients are behind this result: dispersed information, forward-looking agents, and a backward-looking policy based on the observed realization of past shocks. The different information sets of consumers and price setters play a central role in the mechanism described above. The presence of forward-looking agents is clearly needed so that announcements about future policy affect current behavior. The backward-looking policy rule works because it is based on past shock realizations which were not observed by the agents at the time they hit. To clarify this point, notice that the results above would disappear if the central bank based its intervention at $t + 1$ on any variable that is common knowledge at date t , for example on s_t . Suppose, for example, that the backward-looking component of the nominal spending target (9) took the form $\mu_s s_{t-1}$ instead of $\mu_a a_{t-1}$. Then, any adjustment in the

backward-looking parameter μ_s would lead to identical and fully offsetting effects on current prices and expected future prices, with no effects on the real allocation.¹³

4.3 Full aggregate stabilization

Going back to the special case introduced above, it is easy to show that monetary policy can achieve the full information benchmark for aggregate activity by choosing the right value of μ_a . Since $\gamma = 1$, aggregate consumption under full information is $c_t^{fi} = \psi_0 + a_t$, from (10). Equation (13) shows that the central bank can induce the same aggregate outcome by setting $\mu_a = -1$. This monetary policy rule gives, at the same time, $\psi_\theta = 1$ and $\psi_s = 0$.¹⁴ This may seem the outcome of the special case considered and, in particular, of the fact that consumers have full information. In fact, the result holds in general, as shown by the next proposition.

Proposition 4 *There exists a monetary policy rule μ_a^{fs} which, together with the appropriate subsidy τ^{fs} , achieves full aggregate stabilization, that is, an equilibrium with $c_t = c_t^{fi}$.*

To achieve the full information benchmark for c_t , the central bank has to eliminate the effect of noise shocks, setting ψ_s equal to zero, and ensure, at the same time, that the output response to the fundamental shocks ψ_θ is equal to $(1 + \eta) / (\gamma + \eta)$. Given that, by Proposition 3, there is a linear relation between μ_a and ψ_s and $\partial\psi_s/\partial\mu_a \neq 0$, it is always possible to find a μ_a such that ψ_s is equal to zero.¹⁵ The surprising result is that the value of μ_a that sets ψ_s to zero does, at the same time, set ψ_θ equal to $(1 + \eta) / (\gamma + \eta)$. This result is an immediate corollary of the following lemma.

Lemma 2 *In any linear equilibrium, ψ_θ and ψ_s satisfy*

$$\psi_\theta \sigma_\theta^2 + \psi_s \sigma_\epsilon^2 = \frac{1 + \eta}{\gamma + \eta} \sigma_\theta^2.$$

Proof. Starting from the optimal pricing condition (6), take the conditional expectation $\mathbb{E}[\cdot | a_{t-1}, s_t]$ on both sides. Using the law of iterated expectations and the fact that all idiosyncratic shocks have zero mean, yields

$$\mathbb{E} \left[c_t - \psi_0 - \frac{1 + \eta}{\gamma + \eta} a_t | a_{t-1}, s_t \right] = 0. \quad (14)$$

¹³On the other hand, it is not crucial that the central bank observes θ_t perfectly in period $t + 1$. In fact, it is possible to generalize the result above to the case where the central bank receives noisy information about θ_t at $t + 1$, as long as this information is not in the agents' information sets at time t .

¹⁴This does not ensure that ψ_0 will also be the same. However, the subsidy τ can be adjusted to obtain the desired value of ψ_0 .

¹⁵In the proof of Proposition 4, I check that μ_a^{fs} is different from μ_a^0 .

Using (4) to substitute for c_t , this equation boils down to

$$\mathbb{E}[\psi_\theta \theta_t + \psi_s e_t | s_t] = \frac{1 + \eta}{\gamma + \eta} \mathbb{E}[\theta_t | s_t].$$

Substituting for $\mathbb{E}[\theta_t | s_t] = (\sigma_\theta^2 / (\sigma_\theta^2 + \sigma_e^2)) s_t$ and $\mathbb{E}[e_t | s_t] = (\sigma_e^2 / (\sigma_\theta^2 + \sigma_e^2)) s_t$, gives the desired restriction. ■

The point of this lemma is that the output responses to the two shocks are tied together by the price setters' optimality condition. In particular, price setters cannot, based on the public information in a_{t-1} and s_t , expect their prices to deviate systematically from nominal marginal costs plus a constant mark-up. This implies that, conditional on the same information, aggregate consumption cannot be expected to deviate systematically from its full information level, as shown by (14). In turn, this implies that when ψ_θ increases ψ_s must decrease. This also implies that, if aggregate consumption moves one for one with $((1 + \eta) / (\gamma + \eta)) \theta_t$, then the effect of the noise shock e_t must be zero.

To conclude this section, let me remark that the choice of μ_a also affect the sensitivity of individual consumption and prices to idiosyncratic shocks. That is, the policy rule has implications not only for aggregate responses, but also for the cross-sectional distribution of consumption and relative prices. This observation will turn out to be crucial in evaluating the welfare consequences of different monetary rules.

5 Optimal monetary policy

5.1 Welfare

I now turn to welfare analysis and to the characterization of optimal monetary policy. The consumption of good j by consumer i is given by

$$C_{ijt} = P_{jt}^{-\sigma} \bar{P}_{it}^\sigma C_{it}. \quad (15)$$

In a linear equilibrium, using (1), (2) and (5), this expression becomes

$$C_{ijt} = \exp \{ \psi_0 + \sigma \kappa_{\bar{p}} + \psi_a a_{t-1} + \psi_s s_t + \psi_x x_{it} + \psi_{\bar{x}} \bar{x}_{it} - \sigma \phi_x (x_{jt} - \bar{x}_{it}) \}.$$

The equilibrium labor effort of producer i is given by the market clearing condition

$$N_{it} = \frac{\int_{j \in \tilde{J}_{it}} C_{jit} dj}{A_{it}}, \quad (16)$$

where \tilde{J}_{it} denotes the set of consumers who buy good i at time t . Using these expressions and exploiting the normality of the shocks, it is possible to derive analytically the expected utility of a representative household at time 0, as shown in the following lemma.

Lemma 3 Take any monetary policy $\mu_a \in \mathbb{R}/\mu_a^0$ and consider the associated linear equilibrium. Assume the subsidy τ is chosen optimally. Then the expected utility of a representative household is given by

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t U(C_{it}, N_{it}) \right] = \frac{1}{1-\gamma} W_0 e^{(1-\gamma) \frac{1+\eta}{\gamma+\eta} w(\mu_a)},$$

if $\gamma \neq 1$, and by

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t U(C_{it}, N_{it}) \right] = w_0 + \frac{1}{1-\beta} w(\mu_a),$$

if $\gamma = 1$. W_0 and w_0 are constant terms independent of μ_a , W_0 is positive, and $w(\cdot)$ is a known quadratic function, which depends on the model's parameters.

The function $w(\mu_a)$ can be used to evaluate the welfare effects of different policies in terms of equivalent consumption changes. Suppose I want to compare the policies μ'_a and μ''_a by finding the Δ such that

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t U((1+\Delta)C'_{it}, N'_{it}) \right] = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t U(C''_{it}, N''_{it}) \right],$$

where C'_{it}, N'_{it} and C''_{it}, N''_{it} are the corresponding equilibrium allocations. The value of Δ represents the proportional increase in lifetime consumption which is equivalent to a policy change from μ'_a to μ''_a . The following lemma shows that $w(\mu''_a) - w(\mu'_a)$ provides a first-order approximation for Δ .¹⁶

Lemma 4 Let $\Delta(\mu'_a, \mu''_a)$ be the welfare change associated to the policy change from μ'_a to μ''_a , measured in terms of equivalent proportional change in lifetime consumption. The function $\Delta(\cdot, \cdot)$ satisfies

$$\left. \frac{d\Delta(\mu_a, \mu_a + u)}{du} \right|_{u=0} = w'(\mu_a).$$

5.2 Constrained efficiency

To characterize optimal monetary policy, I will show that it achieves an appropriately defined social optimum. I consider a planner who can choose the consumption and labor effort levels C_{ijt} and N_{it} facing only two constraints: the resource constraint (16) and the informational constraint that C_{ijt} be measurable with respect to $a_{t-1}, s_t, x_{it}, \bar{x}_{it}$ and x_{jt} . This requires that, when selecting the consumption basket of consumer i at time t , the planner can only use the

¹⁶I am grateful to Kjetil Storesletten for suggesting this result.

information available to the same consumer in the market economy. Specifically, I allow the planner to use all the information available to consumers in linear equilibria with $\phi_x \neq 0$. Given that the only information on past shocks which is useful to the planner at time t is captured by a_{t-1} , I omit conditioning C_{ijt} on more detailed information on past shocks. An allocation that solves the planner problem is said to be “constrained efficient.”

The crucial assumption here is that the planner can determine how consumers respond to various sources of information, but cannot change this information. This notion of constrained efficiency is developed and analyzed in a broad class of quadratic games in Angeletos and Pavan (2007, 2009). Here, I can apply it in a general equilibrium environment, even though agents extract information from prices, because the matching environment is such that this information is essentially exogenous. The following result shows that, with the right choice of μ_a and τ , the equilibrium found in Proposition 1 is constrained efficient.

Proposition 5 *There exist a monetary policy μ_a^* and a subsidy τ^* such that the associated stationary linear equilibrium is constrained efficient.*

This proposition shows that a simple backward-looking policy rule, which is only contingent on aggregate variables, is sufficient to induce agents to make the best possible use of the public and private information available.

All linear equilibrium allocations are feasible for the planner, as they satisfy both the resource constraint and the measurability constraint. Therefore, an immediate corollary of Proposition 5 is that μ_a^* maximizes $w(\mu_a)$. On the other hand, the set of feasible allocations for the planner is larger than the set of linear equilibrium allocations, because in the planner’s problem C_{ijt} is allowed to be any function, possibly non-linear, of $a_{t-1}, s_t, x_{it}, \bar{x}_{it}$ and x_{jt} .

5.3 Optimal accommodation of noise shocks

Having obtained a general characterization of optimal monetary policy, I can turn to more specific questions: what is the economy’s response to the various shocks at the optimal monetary policy? In particular, is full aggregate stabilization optimal? That is, should monetary policy completely eliminate the aggregate disturbances due to noise shocks, setting $\psi_s = 0$? The next proposition shows that, typically, full aggregate stabilization is suboptimal.

Proposition 6 *Suppose the signal s_t is noisy, $\sigma_e^2 > 0$, and the model parameters satisfy $\eta > 0$ and $\chi \in (0, 1)$, then full aggregate stabilization is suboptimal. If $\sigma\gamma > 1$ then $\mu_a^* < \mu_a^{fs}$ and, at*

the optimal policy, aggregate consumption is less responsive to fundamental shocks than under full information and noise shocks have a positive effect on aggregate consumption:

$$\psi_\theta^* < \frac{1+\eta}{\gamma+\eta}, \quad \psi_s^* > 0.$$

If $\sigma\gamma < 1$ the opposite inequalities apply. Full stabilization is optimal if at least one of the following conditions holds: $\eta = 0$, $\chi = 0$, $\chi = 1$, $\sigma\gamma = 1$.

To interpret this result, I use the following expression for the welfare index $w(\mu_a)$ defined in Lemma 3,

$$\begin{aligned} w = & -\frac{1}{2}(\gamma+\eta)\mathbb{E}[(c_t - c_t^{fi})^2] + \\ & +\frac{1}{2}(1-\gamma)\int_0^1 (c_{it} - c_t)^2 di - \frac{1}{2}(1+\eta)\int_0^1 (n_{it} - n_t)^2 di + (c_t - a_t - n_t), \end{aligned} \quad (17)$$

where n_t is the employment index $n_t \equiv \int_0^1 n_{it} di$. This expression is derived in the appendix. The first term in (17) captures the welfare effects of aggregate volatility. In particular, it shows that social welfare is negatively related to the volatility of the “output gap” measure $c_t - c_t^{fi}$, which captures the distance between c_t and the full-information benchmark analyzed in Section 4.1. A policy of full aggregate stabilization sets this expression to zero. However, the remaining terms are also relevant to evaluate social welfare. Once they are taken into account, full aggregate stabilization is no longer desirable. These terms capture welfare effects associated to the cross-sectional allocation of consumption goods and labor effort, conditional on the aggregate shocks θ_t and e_t . Let me analyze them in order.

The second and third term in (17) capture the effect of the idiosyncratic variances of c_{it} and n_{it} . Since c_{it} and n_{it} are the logs of the original variables, these expressions capture both level and volatility effects. In particular, focusing on the first one, when the distribution of c_{it} is more dispersed, C_{it} is, on average, higher, given that

$$\mathbb{E}[C_{it}|a_{t-1}, \theta_t, e_t] = \exp \left\{ c_t + \frac{1}{2} \int_0^1 (c_{it} - c_t)^2 di \right\},$$

but is also more volatile as

$$\text{Var}[C_{it}|a_{t-1}, \theta_t, e_t] = \exp \left\{ \frac{1}{2} \int_0^1 (c_{it} - c_t)^2 di \right\}.$$

This explains why the term $\int_0^1 (c_{it} - c_t)^2 di$ is multiplied by $1 - \gamma$. When the coefficient of relative risk aversion γ is greater than 1, agents care more about the volatility effect than about the level effect. In this case, an increase in the dispersion of c_{it} reduces consumers’

expected utility. The opposite happens when γ is smaller than 1. A similar argument applies to the third term in (17), although there both the level and the volatility effects reduce expected utility, given that the disutility of effort is a convex function.

The last term, $c_t - a_t - n_t$, reflects the effect of monetary policy on the economy's average productivity in consumption terms. Due to the heterogeneity in consumption baskets, a given average level of labor effort, with given productivities, translates into different levels of the average consumption index c_t depending on the distribution of quantities across consumers and producers. To further analyze this term, I use the following decomposition, which is derived in the appendix,

$$c_t - a_t - n_t = -\frac{1}{2} \text{Var}[c_{jt} + \sigma \bar{p}_{jt} | j \in \tilde{J}_{it}, a_{t-1}, \theta_t, e_t] + \frac{\sigma(\sigma - 1)}{2} \text{Var}[p_{jt} | j \in J_{it}, a_{t-1}, \theta_t, e_t]. \quad (18)$$

To interpret the first term, notice that $c_{jt} + \sigma \bar{p}_{jt}$ is the intercept, in logs, of the demand for good i by consumer j , given by (15). Fixing average log consumption, satisfying a more dispersed log demand requires more effort by producer i . To interpret the second term, notice that consumers like price dispersion in their consumption basket, given that when prices are more variable they can reallocate expenditure from more expensive goods to cheaper ones. Therefore, a given average effort by the producers translates into higher consumption indexes when relative prices are more dispersed.¹⁷ Summing up, when demand dispersion is lower and price dispersion higher, a given average effort n_t generates higher average consumption c_t .

5.4 A numerical example

To illustrate the various welfare effects just described, I turn to a numerical example. The parameters for the example are in Table 1. The coefficient of relative risk aversion γ is set to 1. The values for σ and η are chosen in the range of values commonly used in DSGE models with sticky prices. The values for the variances σ_θ^2 , σ_ϵ^2 and σ_e^2 are set at 1. The variance of the sampling shocks σ_v^2 must then be in $[0, 1]$ and I pick the intermediate value $\sigma_v^2 = 1/2$.

Figure 2 illustrates how μ_a affects the various terms in (17) and compares the optimal policy μ_a^* with the full-stabilization policy μ_a^{fs} . Given that $\gamma\sigma > 1$, the optimal policy is to the left of the full-stabilization policy, by Proposition 6. Using Lemma 4, it is possible to interpret the welfare effects in terms of equivalent consumption changes.

¹⁷Since prices are expressed in logs, an increase in the volatility of p_{jt} has both a level and a volatility effect. Given that $\sigma > 1$, the second always dominates. The fact that relative price dispersion increases welfare is not inconsistent with approximate welfare expressions in standard new Keynesian models, where relative price dispersion enters with a minus sign. The price dispersion term in those expressions summarizes all the cross sectional effects discussed here, including, in particular, the dispersion of labor supply. Due to heterogenous productivity, this simplification is not possible in the model presented here.

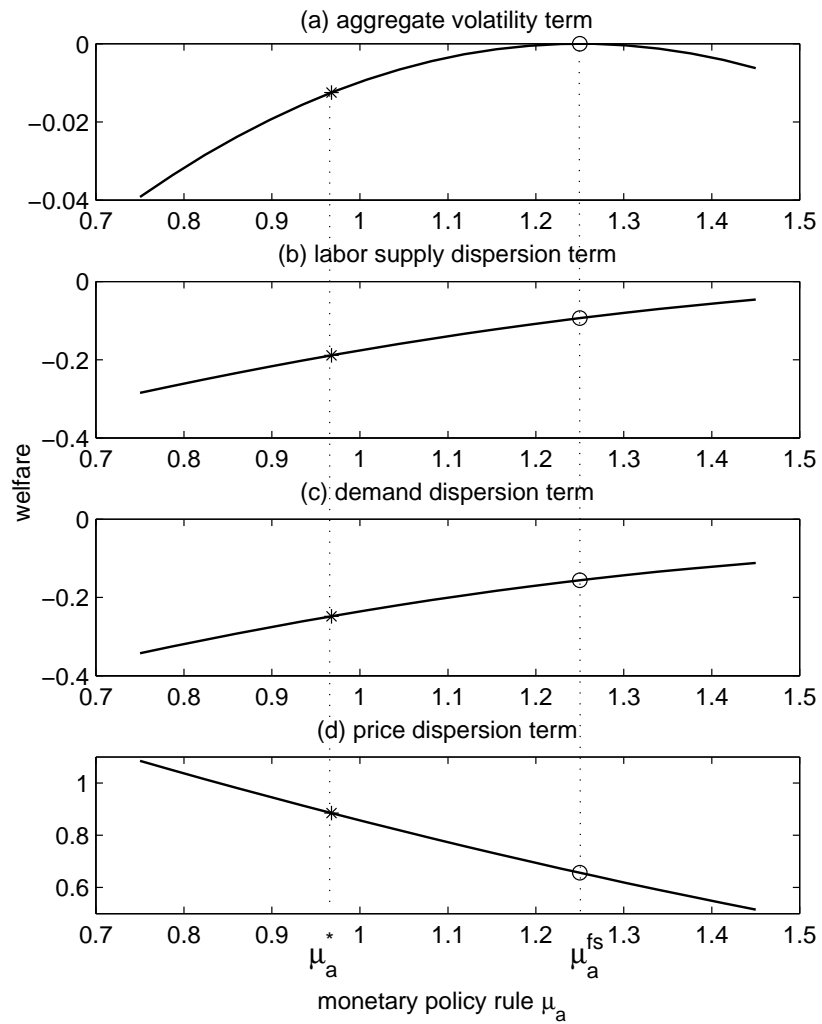


Figure 2: Decomposing the welfare effects of monetary policy.

γ	1	η	2
σ	7		
σ_θ^2	1	σ_ϵ^2	1
σ_e^2	1	σ_v^2	0.5

Table 1: Parameters for the numerical example.

Panel (a) plots the relation between μ_a and the first term in (17), capturing the negative effect of aggregate volatility. Not surprisingly, the maximum of this function is reached at the full-stabilization policy. Focusing purely on the aggregate output gap, the social planner finds that moving from μ_a^{fs} to μ_a^* leads to an approximate welfare loss of 1% in equivalent consumption. However, when all cross-sectional terms are taken into account, the same policy change generates, in fact, a welfare gain of about 3%. Although this is just an example, these numbers show that disregarding the cross-sectional implications of monetary policy can lead to serious welfare miscalculations.

Let me now examine the cross-sectional terms in more detail. With $\gamma = 1$, the second term in (17) is always zero, so this term is not reported in the figure. Panel (b) plots the third term, the negative effect due to the dispersion in labor supply. Panels (c) and (d) plot separately the two components of the productivity term $c_t - a_t - n_t$, derived in equation (18): the demand dispersion term in panel (c) and the price dispersion term in panel (d).

Notice the crucial role of the price dispersion term. Moving from μ_a^{fs} to μ_a^* leads to a welfare loss of about 9% in terms of labor supply dispersion and to a similar loss in terms of demand dispersion, as shown in panels (b) and (c). The welfare gain due to increased price dispersion is very large, about 22%, and more than compensates for these losses and for the aggregate volatility loss in panel (a). Let me provide some intuition for the mechanism behind these effects.

At the optimal equilibrium, ϕ_x is negative: producers with higher productivity set lower prices to induce consumers to buy more of their goods.¹⁸ By increasing μ_a , the central bank induces household consumption to be more responsive to the private productivity signal x_{it} .¹⁹ This implies that more productive households face lower marginal utility of consumption, and, at the price-setting stage, have weaker incentives to lower prices. In formal terms, $\partial\phi_x/\partial\mu_a > 0$, as shown in Proposition 3. Therefore, increasing μ_a in a neighborhood of μ_a^* , reduces $|\phi_x|$, the absolute response of prices to individual productivity shocks and causes relative prices to be

¹⁸See the proof of Proposition 5.

¹⁹Equation (45), in the appendix, implies $\partial\psi_x/\partial\mu_a > 0$.

less responsive to individual productivity differences. This leads to a more compressed price distribution and to the welfare loss depicted in panel (d).

Under the parametric assumptions made, this mechanism also leads to a reduction in labor supply dispersion and in demand dispersion, as shown in panels (b) and (c). In the example, at μ_a^* , individual labor supply is increasing in individual productivity. When relative prices become less responsive to individual productivity, the relation between productivity and labor supply becomes flatter, reducing the cross-sectional dispersion in labor supply. Finally, an increase in μ_a leads to a compression in the distribution of demand indexes faced by a given producer, due to the reduced dispersion in the price indexes \bar{p}_{jt} .

Summing up, if the central bank wants to reach full stabilization it has to induce households to rely more heavily on their private productivity signals x_{it} when making their consumption decisions. By inducing them to concentrate on private signals the central bank can mute the effect of public noise. However, in doing so, the central bank reduces the sensitivity of individual prices to productivity, generating an inefficiently compressed distribution of relative prices.

5.5 The role of strategic complementarity in pricing

Proposition 6 identifies a set of special cases where full stabilization is optimal. An especially interesting case is $\eta = 0$. In this case, there is no strategic complementarity in price setting. Substituting the consumer's Euler equation (7) on the right-hand side of the pricing condition (6) and using the law of iterated expectations, after some manipulations, yields

$$p_{it} = \mu_a a_{t-1} + (\mu_a + \rho) \mathbb{E}_{i,(t,I)} [\theta_t] - x_{it}. \quad (19)$$

This shows that in this case prices only depend on the agents' first-order expectations of θ_t . The analysis in Section 4.2 shows that even in this simple case an interesting form of non-neutrality is present, because of asymmetric information between price setters and consumers. However, in this case there is no significant interaction among price setters. That is, the strategic complementarity emphasized in Woodford (2002) and Hellwig (2005) is completely muted.

In this case, it can be shown that the optimal monetary policy is $\mu_a^* = -\rho$. The consumer's Euler equation implies that under this policy the marginal utility of expenditure $\bar{P}_{it}^{-1} C_{it}^{-\gamma}$ is perfectly equalized across households. At the same time, by (19), relative prices are perfectly proportional to individual productivities. These relative prices achieve an efficient cross-sectional

allocation of consumption and labor effort. That is, in this economy there is no tension between aggregate and cross-sectional efficiency. Actually, it is possible to prove that, at the optimal monetary policy, this economy achieves the full-information first-best allocation.²⁰

When $\eta \neq 0$, producers must forecast their sales to set optimal prices and these sales depend on the prices set by other producers. Now the pricing decisions of the producers are fully interdependent. On the planner's front, when $\eta \neq 0$, it is necessary to use individual estimates of θ_t when setting efficient "shadow" prices. In this case, the optimal policy can no longer achieve the unconstrained first-best. Therefore, the presence of strategic complementarity in pricing is tightly connected to the presence of an interesting trade-off between aggregate and cross-sectional efficiency.

6 The welfare effects of public information

So far, I have assumed that the source of public information, the signal s_t , is exogenous and outside the control of the monetary authority. Suppose now that the central bank has some control on the information received by the private sector. For example, it can decide whether or not to systematically release some aggregate statistics, which would increase the precision of public information. What are the welfare effects of this decision? To address this question I look at the effects of changing the precision of the public signal, defined as $\pi_s \equiv 1/\sigma_e^2$, on total welfare. This exercise connects this paper to the growing literature on the welfare effects of public information, discussed in the introduction. I consider two possible versions of this exercise. First, I assume that when π_s changes the monetary policy rule μ_a is kept constant, while the subsidy τ is adjusted to its new optimal level. Second, I assume that for each value of π_s both μ_a and τ are chosen optimally.

Suppose the economy's parameters are those in Table 1 and suppose that μ_a is fixed at its optimal value for $\pi_s = 1$. Figure 3 shows the effect of changing π_s on welfare. The solid line represents total welfare, measured by w in (17). The dashed line represents the relation between μ_a and the first component in (17), which captures the welfare cost of aggregate volatility. To improve readability, welfare measures are expressed in terms of differences from their value at $\pi_s = 1$ and a log scale is used for π_s . Let me begin by discussing the second relation. When the signal s_t is very imprecise agents disregard it and the coefficient ψ_s goes to zero. When the

²⁰To prove this, follow the same steps as in the proof of Proposition 5, but allow the consumption rule to be contingent on θ_t . Then, it is possible to show that the optimal allocation is supported by the equilibrium described above.

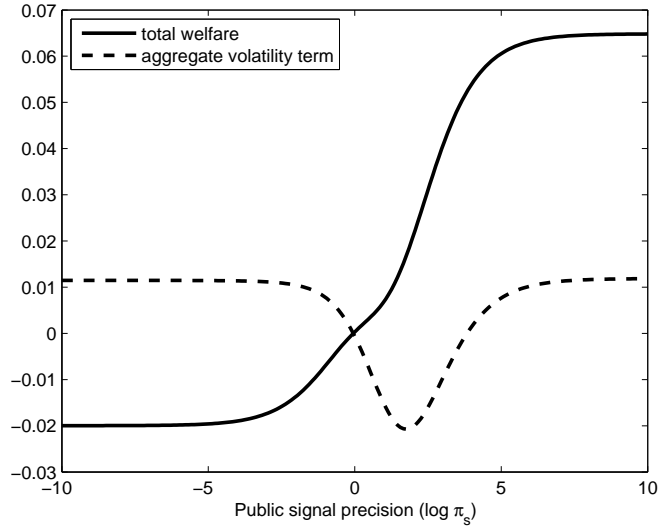


Figure 3: Welfare effects of changing the public signal precision, $\eta = 2$.

signal becomes more precise, agents rely more on the public signal. So, although the volatility of e_t is falling, increasing ψ_s can lead to an increase in aggregate volatility. In the example considered, this happens whenever $\log \pi_s$ is below 1.8. In that region, more precise public information has a destabilizing effect on the economy. Eventually, when the signal precision is sufficiently large, the economy converges towards the full information equilibrium and output gap volatility goes to zero. Therefore, there is a non-monotone relation between μ_a and the cost of aggregate volatility. However, this only captures the first piece of the welfare function (17). The solid line in Figure 3 shows that, when all the other pieces are taken into account, welfare is increasing everywhere in π_s . When the public signal is very imprecise, agents have to use their own individual productivity to estimate aggregate productivity. This leads to less precise estimates of idiosyncratic productivity, leading to a compressed distribution of relative prices. An increase in the signal precision helps producers set relative prices that reflect more closely the underlying productivity differentials. The associated gain in allocative efficiency is always positive and more than compensates for the potential welfare losses due to higher aggregate volatility.

The notion that more precise information about aggregate variables has important cross-sectional implications is also highlighted in Hellwig (2005). In that paper, agents face uncertainty about monetary policy shocks and there are no idiosyncratic productivity shocks. Therefore, the cross-sectional benefits of increased transparency are reflected in a reduction in

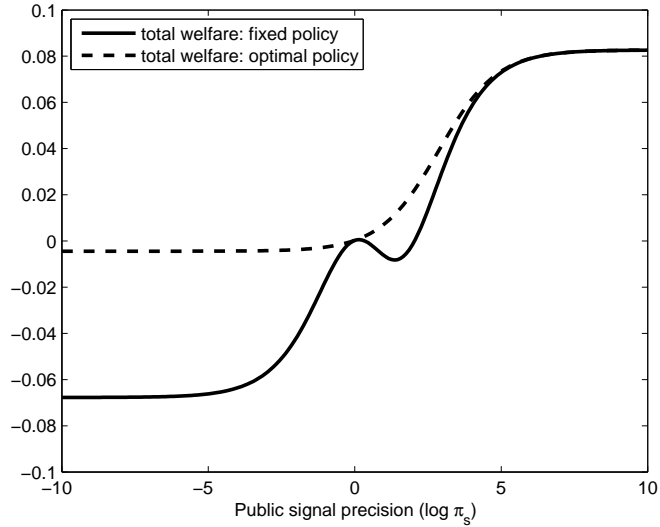


Figure 4: Welfare effects of changing the public signal precision, $\eta = 5$.

price dispersion. Here, instead, more precise public information tends to generate higher price dispersion. However, the underlying principle is the same: in both cases a more precise public signal leads to relative prices more in line with productivity differentials.

Let me now consider a more intriguing example, where social welfare can be decreasing in π_s . In Figure 4, I plot the relation between π_s and w for an economy identical to the one above, except that the inverse Frisch elasticity of labor supply is set to a much higher value, $\eta = 5$. When η is larger, the costs of aggregate volatility are bigger, and, it is possible to have a non-monotone relationship between π_s and total welfare, as shown by the solid line in Figure 4. For example, when $\log \pi$ increases from 0 to 1, social welfare falls by about 0.6% in consumption equivalent terms. This result mirrors the result obtained by Morris and Shin (2002) in a simple quadratic game. As stressed by Angeletos and Pavan (2007), their result depends crucially on the form of the agents' objective function and on the nature of their strategic interaction. In my model, the possibility of welfare-decreasing public information depends on the balance between aggregate and cross-sectional effects. When η is large the negative welfare effects of aggregate volatility become a dominant concern, and increases in public signal precision can be undesirable.

This result disappears when I allow the central bank to adjust the monetary policy rule to changes in the informational environment. In this case, more precise public information is unambiguously good for social welfare. This is illustrated by the dashed line in Figure 4,

which shows the relation between π_s and w , when μ_a is chosen optimally. By Proposition 5, the optimal μ_a induces agents to use information in a socially optimal way. Therefore, at the optimal policy, better information always leads to higher social welfare. The underlying argument is analogous to that used by Angeletos and Pavan (2007) in the context of quadratic coordination games: when the equilibrium is constrained efficient more precise information is always welfare improving.

Proposition 7 *If μ_a is kept fixed, an increase in π_s can lead to a welfare gain or to a welfare loss, depending on the model's parameters. If μ_a is chosen optimally, increasing π_s is always (weakly) welfare improving.*

7 Conclusions

In this paper, I have explored the role of monetary policy rules in an economy where information about macroeconomic fundamentals is dispersed across agents. The emphasis has been on the ability of the policy rule to shape the economy's response to different shocks. In particular, the monetary authority is able to reduce the economy's response to noise shocks by manipulating agents' expectations about the real interest rate. The principle behind this result goes beyond the specific model used in this paper: by announcing that policy actions will respond to future information, the monetary authority can affect differently agents with different pieces of information. In this way, it can change the aggregate response to fundamental and noise shocks even if it has no informational advantage over the private sector. A second general lesson that comes from the model is that, in the presence of heterogeneity and dispersed information, the policy maker will typically face a trade-off between aggregate and cross-sectional efficiency. Inducing agents to be more responsive to perfectly observed local information can lead to aggregate outcomes that are less sensitive to public noise shocks, but it can also lead to a worse cross-sectional allocation.

The optimal policy rule used in this paper can be implemented both under commitment and under discretion. To offset an expansion driven by optimistic beliefs, the central bank announces that it will make the realized real interest rate higher if good fundamentals do not materialize. With flexible prices, this effect is achieved with a downward jump in the price level between t and $t+1$. Since a_t is common knowledge at time $t+1$, this jump only affects nominal variables, but has no consequences on the real allocation in that period. Therefore, the central bank has no incentive to deviate ex post from its announced policy. In economies with sluggish

price adjustment, a similar effect could be obtained by a combination of a price level change and an increase in nominal interest rates. In that case, however, commitment problems are likely to arise, because both type of interventions have additional distortionary consequences ex post. The study of models where lack of commitment interferes with the central bank's ability to deal with informational shocks is an interesting area for future research.

A strong simplifying assumption in the model is that the only financial assets traded in subperiods (t, I) and (t, II) are non-state-contingent claims on dollars at $(t+1, 0)$. Introducing a richer set of traded financial assets would increase the number of price signals available to both consumers and the monetary authority. In a simple environment with only two aggregate shocks, this will easily lead to a fully revealing equilibrium. Therefore, to fruitfully extend the analysis in this direction requires the introduction of a larger number of shocks, making financial prices noisy indicators of the economy's fundamentals.

Finally, in the model presented, the information available to the central bank is independent of the policy rule, as all aggregate shocks are fully revealed after one period. Morris and Shin (2005) have recently argued that stabilization policies may have adverse effects, if they reduce the informational content of prices for policy makers. A natural extension of the model in this paper would be to enrich the informational dynamics, making the information available to the central bank endogenous and sensitive to policy.

8 Appendix

8.1 Random consumption baskets

At the beginning of each period, household i is assigned two random variables, ϵ_{it} and v_{it} , independently drawn from normal distributions with mean zero and variances, respectively, σ_ϵ^2 and σ_v^2 . These variables are not observed by the household. The first random variable represents the idiosyncratic productivity shock, the second is the sampling shock that will determine the sample of firms visited by consumer i . Consumers and producers are then randomly matched so that: (i) each consumer meets a fixed mass $M < 1$ of producers and each producer a fixed mass M of consumers; and (ii) the mass of matches between producers with productivity shock ϵ and consumers with sampling shock v is $M\phi(\epsilon, v)$, where $\phi(\epsilon, v)$ is the bivariate normal density with covariance matrix

$$\begin{bmatrix} \sigma_\epsilon^2 & \sqrt{\chi}\sigma_v\sigma_\epsilon \\ \cdot & \sigma_v^2 \end{bmatrix}$$

and χ is a parameter in $[0, 1]$. Since the variable v_{it} has no direct effect on payoffs, its variance is normalized and set to $\sigma_v^2 = \chi\sigma_\epsilon^2$. Let J_{it} denote the set of producers met by consumer i and \tilde{J}_{it} the set of consumers met by producer i . Given the matching process above the following properties follow. The distribution $\{\epsilon_{jt} : j \in J_{it}\}$ is a normal $N(v_{it}, \sigma_{\epsilon|v}^2)$ with $\sigma_{\epsilon|v}^2 = (1 - \chi)\sigma_\epsilon^2$. The distribution $\{v_{jt} : j \in \tilde{J}_{it}\}$ is a normal $N(\chi\epsilon_{it}, \sigma_{v|\epsilon}^2)$, with $\sigma_{v|\epsilon}^2 = \chi(1 - \chi)\sigma_\epsilon^2$.

8.2 Proof of Proposition 1

The proof is split in steps. First, I derive price and demand indexes that apply in the linear equilibrium conjectured. Second, I use them to setup the individual optimization problem and derive necessary conditions for individual optimality. Third, I use these conditions to characterize a linear equilibrium. Fourth, I show how choosing μ_a uniquely pins down the coefficients $\{\phi, \psi\}$ and derive the remaining coefficients of the monetary policy rule that implements $\{\phi, \psi\}$. The proof of local determinacy is in the supplementary material.

8.2.1 Price and demand indexes

Individual optimization implies that the consumption of good j by consumer i is $C_{ijt} = P_{jt}^{-\sigma} \bar{P}_{it}^\sigma C_{it}$, where \bar{P}_{it} is the price index

$$\bar{P}_{it} = \left(\int_{j \in J_{it}} P_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}.$$

The demand for good i is then obtained by integrating the individual demands over \tilde{J}_{it} (the set of consumers who buy good i at time t). This gives $Y_{it} = D_{it}P_{it}^{-\sigma}$, where D_{it} is the “demand index”

$$D_{it} = \int_{j \in \tilde{J}_{it}} \bar{P}_{jt}^\sigma C_{jt} dj.$$

The next lemma derives explicit expressions for price and demand indexes in a linear equilibrium.

Lemma 5 *If individual prices and quantities are given by (1) and (2) then the price index for consumer i and the demand index for producer i are equal to (5) and*

$$d_{it} = \kappa_d + c_t + \sigma p_t + (\psi_{\bar{x}} + \sigma \phi_x) \chi \epsilon_{it}, \quad (20)$$

where $\kappa_{\bar{p}}$ and κ_d are constant terms equal to

$$\kappa_{\bar{p}} = \frac{1 - \sigma}{2} \phi_x^2 \sigma_\epsilon^2, \quad (21)$$

$$\kappa_d = \frac{1}{2} \psi_x^2 \sigma_\epsilon^2 + \frac{1}{2} (\psi_{\bar{x}} + \sigma \phi_x)^2 \sigma_{v|\epsilon}^2 + \sigma \frac{1 - \sigma}{2} \phi_x^2 \sigma_{\epsilon|v}^2. \quad (22)$$

Proof. Recall that the shocks ϵ_{jt} for $j \in J_{it}$ have a normal distribution $N(v_{it}, \sigma_{\epsilon|v}^2)$. Then, given (1), the prices observed by consumer i are log-normally distributed, with mean $p_t + \phi_x v_{it}$ and variance $\phi_x^2 \sigma_{\epsilon|v}^2$, therefore,

$$\int_{j \in J_{it}} P_{jt}^{1-\sigma} dj = e^{(1-\sigma)(p_t + \phi_x v_{it}) + \frac{(1-\sigma)^2}{2} \phi_x^2 \sigma_{\epsilon|v}^2}.$$

Taking both sides to the power $1/(1 - \sigma)$ gives the desired expression for \bar{P}_{it} , from which (5) and (21) follow immediately. Using this result and expression (2), the demand index for producer i can be written as

$$D_{it} = \int_{j \in \tilde{J}_{it}} C_{jt} \bar{P}_{jt}^\sigma dj = e^{c_t + \sigma p_t} e^{\sigma \kappa_{\bar{p}}} \int_{j \in \tilde{J}_{it}} e^{\psi_x \epsilon_{jt} + \psi_{\bar{x}} v_{jt}} e^{\sigma \phi_x v_{jt}} dj.$$

Recall that the distribution $\{v_{jt} : j \in \tilde{J}_{it}\}$ is a normal $N(\chi \epsilon_{it}, \sigma_{v|\epsilon}^2)$, and ϵ_{jt} and v_{jt} are independent. It follows that

$$\int_{j \in \tilde{J}_{it}} e^{\psi_x \epsilon_{jt} + \psi_{\bar{x}} v_{jt} + \sigma \phi_x v_{jt}} dj = e^{\frac{1}{2} \psi_x^2 \sigma_\epsilon^2 + (\psi_{\bar{x}} + \sigma \phi_x) \chi \epsilon_{it} + \frac{1}{2} (\psi_{\bar{x}} + \sigma \phi_x)^2 \sigma_{v|\epsilon}^2}.$$

Substituting in the previous expression gives (20) and (22). ■

8.2.2 Individual optimization

Consider an individual household, who expects all other households to follow (1)-(2) and the central bank to follow (8)-(9). In period $(t, 0)$, before all current shocks are realized, the household's expectations about the current and future path of prices, quantities and interest rates depend only on a_{t-1} and R_t . Moreover, the only relevant individual state variable is given by the household nominal balances B_{it} . Therefore, I can analyze the household's problem using the Bellman equation

$$V(B_{it}, a_{t-1}, R_t) = \max_{\substack{\{Z_{it+1}(\cdot)\}, \{B_{it+1}(\cdot)\}, \\ \{P(\cdot, \cdot)\}, \{C(\cdot, \cdot, \cdot)\}}} \mathbb{E}_t [U(C_{it}, N_{it}) + \beta V(B_{it+1}, a_t, R_{t+1})]$$

subject to the constraints

$$B_{it+1}(\omega_{it}) = R_t \left[B_{it} - \int q(\tilde{\omega}_{it}) Z_{it+1}(\tilde{\omega}_{it}) d\tilde{\omega}_{it} + (1 + \tau) P_{it} Y_{it} - \bar{P}_{it} C_{it} - T_t \right] + Z_{it+1}(\omega_{it}),$$

$$Y_{it} = D_{it} P_{it}^{-\sigma}, \quad Y_{it} = A_{it} N_{it}, \quad P_{it} = P(a_{t-1}, s_t, x_{it}), \quad C_{it} = C(a_{t-1}, s_t, x_{it}, \bar{x}_{it}),$$

and the law of motions for a_t and R_{t+1} . $E_t[\cdot]$ represents expectations formed at $(t, 0)$ and, in the equilibrium conjectured, it can be replaced by $E[\cdot | a_{t-1}]$. This problem gives the following optimality conditions for prices and consumption

$$\mathbb{E}_{i,(t,I)} \left[(1 + \tau) \bar{P}_{it}^{-1} C_{it}^{-\gamma} Y_{it} - \frac{\sigma}{\sigma - 1} A_{it}^{-1} N_{it}^\eta P_{it}^{-1} Y_{it} \right] = 0, \quad (23)$$

$$\mathbb{E}_{i,(t,II)} \left[\bar{P}_{it}^{-1} C_{it}^{-\gamma} - \beta R_t \bar{P}_{it+1}^{-1} C_{it+1}^{-\gamma} \right] = 0, \quad (24)$$

where $E_{i,(t,I)}[\cdot]$ and $E_{i,(t,II)}[\cdot]$ denote the expectations of agent i at (t, I) and (t, II) . Given the conjectured equilibrium and, given Lemma 1, they are equal to $E[\cdot | a_{t-1}, s_t, x_{it}]$ and $E[\cdot | a_{t-1}, s_t, x_{it}, \phi_x \bar{x}_{it}]$. By Lemma 5 all the random variables in the expressions above are log-normal, including the output and labor supply of producer i which are equal to $Y_{it} = D_{it} P_{it}^{-\sigma}$ and $N_{it} = A_{it}^{-1} Y_{it}$. Rearranging and substituting in (23) and (24) gives (6) and (7) in the text, which I report here in extended form,

$$p_{it} = \kappa_p + \eta (\mathbb{E}_{i,(t,I)} [d_{it}] - \sigma p_{it} - a_{it}) + \mathbb{E}_{i,(t,I)} [\bar{p}_{it} + \gamma c_{it}] - a_{it}, \quad (25)$$

$$\bar{p}_{it} + \gamma c_{it} = \gamma \kappa_c - r_t + \mathbb{E}_{i,(t,II)} [\bar{p}_{it+1} + \gamma c_{it+1}]. \quad (26)$$

The constant terms κ_p and κ_c are equal to

$$\kappa_p = H(\psi_s, \psi_x, \psi_{\bar{x}}, \phi_x) - \log(1 + \tau), \quad (27)$$

$$\kappa_c = G(\psi_s, \psi_x, \psi_{\bar{x}}, \phi_x), \quad (28)$$

and H and G are known quadratic functions of $\psi_s, \psi_x, \psi_{\bar{x}}$ and ϕ_x .

8.2.3 Equilibrium characterization

To check for individual optimality, I will substitute the conjectures made for individual behavior, (1) and (2), in the optimality conditions (25) and (26) and obtain a set of restrictions on $\{\phi, \psi\}$. Notice that all the shocks are i.i.d. so the expected value of all future shocks is zero. Let me assume for now that $\phi_x \neq 0$, so that $E[\cdot|s_t, x_{it}, \bar{x}_{it}]$ can replace $E[\cdot|s_t, x_{it}, \phi_x \bar{x}_{it}]$. Let β_s, β_x and $\delta_s, \delta_x, \delta_{\bar{x}}$ be coefficients such that $E[\theta_t|s_t, x_{it}] = \beta_s s_t + \beta_x x_{it}$ and $E[\theta_t|s_t, x_{it}, \bar{x}_{it}] = \delta_s s_t + \delta_x x_{it} + \delta_{\bar{x}} \bar{x}_{it}$. Defining the precision parameters $\pi_\theta \equiv (\sigma_\theta^2)^{-1}$, $\pi_s \equiv (\sigma_e^2)^{-1}$, $\pi_x \equiv (\sigma_\epsilon^2)^{-1}$, and $\pi_{\bar{x}} \equiv (\sigma_v^2)^{-1}$, the coefficients β_s, β_x and $\delta_s, \delta_x, \delta_{\bar{x}}$ are

$$\beta_s = \frac{\pi_s}{\pi_\theta + \pi_s + \pi_x}, \quad \beta_x = \frac{\pi_x}{\pi_\theta + \pi_s + \pi_x}, \quad (29)$$

$$\delta_s = \frac{\pi_s}{\pi_\theta + \pi_s + \pi_x + \pi_{\bar{x}}}, \quad \delta_x = \frac{\pi_x}{\pi_\theta + \pi_s + \pi_x + \pi_{\bar{x}}}, \quad \delta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\pi_\theta + \pi_s + \pi_x + \pi_{\bar{x}}}. \quad (30)$$

I use (5) and (20) to substitute for \bar{p}_{it} and d_{it} in the optimality conditions (25) and (26), and then I use (1)-(4) to substitute for p_{it} , c_{it} , p_t and c_t . Finally, I use $\epsilon_{it} = x_{it} - \theta_{it}$ and $v_{it} = \bar{x}_{it} - \theta_{it}$, and I substitute for $E[\theta_t|s_t, x_{it}]$ and $E[\theta_t|s_t, x_{it}, \bar{x}_{it}]$. After these substitutions, (25) and (26) give two linear equations in $a_{t-1}, s_t, x_{it}, \bar{x}_{it}$. Matching the coefficients term by term and rearranging gives me the following set of equations.

$$(\gamma + \eta)\psi_0 = -(\kappa_p + \kappa_{\bar{p}} + \eta\kappa_d), \quad (31)$$

$$(\gamma + \eta)\psi_a = (1 + \eta)\rho, \quad (32)$$

$$r_t = \gamma\kappa_c - (\phi_a + \gamma\psi_a)(1 - \rho)a_{t-1}, \quad (33)$$

$$(1 + \eta\sigma)\phi_s = \eta(\psi_s + \sigma\phi_s + (\psi_x + \psi_{\bar{x}} + \sigma\phi_x)\beta_s - (\psi_{\bar{x}} + \sigma\phi_x)\chi\beta_s) + (\phi_a + \gamma\psi_a)\beta_s, \quad (34)$$

$$(1 + \eta\sigma)\phi_x = \eta((\psi_x + \psi_{\bar{x}} + \sigma\phi_x)\beta_x + (\psi_{\bar{x}} + \sigma\phi_x)\chi(1 - \beta_x) - 1) + (\phi_a + \gamma\psi_a)\beta_x - 1, \quad (35)$$

$$\phi_s + \gamma\psi_s = (\phi_a + \gamma\psi_a)\delta_s, \quad (36)$$

$$\gamma\psi_x = (\phi_a + \gamma\psi_a)\delta_x, \quad (37)$$

$$\phi_x + \gamma\psi_{\bar{x}} = (\phi_a + \gamma\psi_a)\delta_{\bar{x}}. \quad (38)$$

Notice that ψ_a is given immediately by (32) and is independent of all other parameters. Next, notice that to ensure that (33) holds in equilibrium for any choice of $\xi_p \in R$, the following

conditions need to be satisfied

$$\mu_a = \phi_a, \quad (39)$$

$$\mu_\theta = \phi_s + \phi_x, \quad (40)$$

$$\mu_e = \phi_s, \quad (41)$$

$$\xi_0 = \gamma \kappa_c, \quad (42)$$

$$\xi_a = -(\phi_a + \gamma \psi_a)(1 - \rho), \quad (43)$$

8.2.4 Constructing the linear equilibrium for given μ_a

To simplify notation, I define

$$\tilde{\mu} \equiv \mu_a + \gamma \psi_a,$$

and derive the values of the remaining parameters in $\{\phi, \psi\}$ as a function of $\tilde{\mu}$. Substituting ϕ_a with μ_a (from (39)) in (35), (37) and (38) and rearranging, yields

$$\phi_x = \frac{(\beta_x + \eta \gamma^{-1} (\delta_x \beta_x + \delta_{\bar{x}} (\beta_x + \chi (1 - \beta_x)))) \tilde{\mu} - 1 - \eta}{1 + \eta \sigma - \eta (\sigma - \gamma^{-1}) (\beta_x + \chi (1 - \beta_x))}, \quad (44)$$

$$\psi_x = \gamma^{-1} \delta_x \tilde{\mu}, \quad (45)$$

$$\psi_{\bar{x}} = \gamma^{-1} (\delta_{\bar{x}} \tilde{\mu} - \phi_x). \quad (46)$$

Note that a solution for ϕ_x always exists since $1 + \eta \sigma - \eta (\sigma - \gamma^{-1}) (\beta_x + \chi (1 - \beta_x)) > 0$. This inequality follows from $\beta_x \in [0, 1]$ and $\chi \in [0, 1]$. Next, combining (34) and (36), and using (44)-(46), yields

$$\begin{aligned} \phi_s &= \frac{(\beta_s + \eta \gamma^{-1} \delta_s + \eta \gamma^{-1} (\delta_x + \delta_{\bar{x}} (1 - \chi)) \beta_s) \tilde{\mu} + \eta (\sigma - \gamma^{-1}) (1 - \chi) \phi_x \beta_s}{1 + \eta \gamma^{-1}}, \\ \psi_s &= \gamma^{-1} (\delta_s \tilde{\mu} - \phi_s). \end{aligned} \quad (47)$$

Substituting the values of $\psi_s, \psi_x, \psi_{\bar{x}}$ and ϕ_x thus obtained in (21), (22), (27), gives the equilibrium values of $\kappa_{\bar{p}}, \kappa_d$ and κ_p . Substituting these values in (31), shows that ψ_0 takes the form

$$\psi_0 = J(\psi_s, \psi_x, \psi_{\bar{x}}, \phi_x) + \frac{\log(1 + \tau)}{\gamma + \eta}, \quad (48)$$

where J is a known quadratic function of $\psi_s, \psi_x, \psi_{\bar{x}}$ and ϕ_x . To find the remaining parameters of the monetary policy rule, use (40)-(43).

Next, I check that the market for state-contingent claims clears and that B_{it} is constant and equal to 0. Let $f(\epsilon_{it}, v_{it}, \theta_t, e_t)$ denote the joint density of the shocks ϵ_{it} , v_{it} , θ_t , and e_t . Recall that $\omega_{it} \equiv (\epsilon_{it}, v_{it}, \theta_t, e_t)$ and let the prices of state-contingent claims at $(t, 0)$ be

$$Q(\omega_{it}) = R_t^{-1} f(\epsilon_{it}, v_{it}, \theta_t, e_t) g(\theta_t), \quad (49)$$

where $g(\cdot)$ is a function to be determined. Suppose $B_{it} = 0$. Let the portfolio of state-contingent claims be the same for each household and equal to

$$Z_{it+1}(\omega_{it}) = R_t [\bar{P}_{it} C_{it} - (1 + \tau) Y_{it} P_{it} + T_t].$$

For each realization of the aggregate shocks θ_t and e_t , goods markets clearing and the government budget balance condition imply that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_{it+1}(\{\epsilon, v, \theta_t, e_t\}) f(\epsilon, v, \theta_t, e_t) d\epsilon dv = R_t \int_0^1 (P_{it} Y_{it} - \bar{P}_{it} C_{it}) di = 0.$$

This implies that the market for state-contingent claims clears for each aggregate state θ_t . It also implies that the portfolio $\{Z_{it+1}(\omega_{it})\}$ has zero value at date $(t, 0)$ given that

$$\int_{\mathbb{R}^4} Q(\omega_{it}) Z_{it+1}(\omega_{it}) d\omega_{it} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Z_{it+1}(\{\epsilon, v, \theta, e\}) f(\epsilon, v, \theta, e) g(\theta) d\epsilon dv d\theta de = 0.$$

Substituting in the household budget constraint shows that $B_{it+1} = 0$. Let me check that the portfolio just described is optimal. The first-order conditions for $Z_{it+1}(\omega_{it})$ and $B_{it+1}(\omega_{it})$ are, respectively,

$$\begin{aligned} \lambda(\omega_{it}) &= R_t Q(\omega_{it}) \int_{\mathbb{R}^4} \lambda(\tilde{\omega}_{it}) d\tilde{\omega}_{it}, \\ \lambda(\omega_{it}) &= \frac{\partial V(0, a_t, R_{t+1})}{\partial B_{it+1}} f(\omega_{it}), \end{aligned}$$

where $\lambda(\omega_{it})$ is the Lagrange multiplier on the budget constraint. Combining them and substituting for $\partial V / \partial B_{it+1}$, using the envelope condition

$$\frac{\partial V(0, a_t, R_{t+1})}{\partial B_{it+1}} = \mathbb{E} \left[\bar{P}_{it+1}^{-1} C_{it+1}^{-\gamma} | a_t \right],$$

yields

$$\mathbb{E} \left[\bar{P}_{it+1}^{-1} C_{it+1}^{-\gamma} | a_t \right] f(\omega_{it}) = R_t Q(\omega_{it}) \int_{-\infty}^{\infty} \mathbb{E} \left[\bar{P}_{it+1}^{-1} C_{it+1}^{-\gamma} | \rho a_{t-1} + \tilde{\theta} \right] f(\tilde{\theta}) d\tilde{\theta}.$$

Substituting (1), (2), and (49), and eliminating the constant factors on both sides, this becomes

$$e^{-(\phi_a + \gamma \psi_a)(\rho a_{t-1} + \theta_t)} = g(\theta_t) \int_{-\infty}^{\infty} e^{-(\phi_a + \gamma \psi_a)(\rho a_{t-1} + \tilde{\theta})} f(\tilde{\theta}) d\tilde{\theta},$$

which is satisfied as long as the function $g(\cdot)$ is given by

$$g(\theta_t) \equiv \exp \left\{ -(\phi_a + \gamma\psi_a)\theta_t - \frac{1}{2}(\phi_a + \gamma\psi_a)^2 \sigma_\theta^2 \right\}.$$

Finally, to complete the equilibrium construction, I need to check that $\phi_x \neq 0$. From (44), this requires $\mu_a \neq \mu_a^0$ where

$$\mu_a^0 \equiv \frac{1 + \eta}{\beta_x + \eta\gamma^{-1}(\delta_x\beta_x + \delta_{\bar{x}}(\beta_x + \chi(1 - \beta_x)))} - \rho\gamma \frac{1 + \eta}{\gamma + \eta}.$$

Notice that when $\mu_a = \mu_a^0$ a stationary linear equilibrium fails to exist. A stationary equilibrium with $\phi_x = 0$ can arise, but under a policy μ_a typically different from μ_a^0 . If $\phi_x = 0$ all the derivations above go through, except that $E[\theta_t | s_t, x_{it}, \phi_x \bar{x}_{it}] = \beta_s s_t + \beta_x x_{it}$. Therefore, it is possible to derive the analogous of condition (44) and show that $\phi_x = 0$ iff

$$\frac{(1 + \eta\gamma^{-1}\beta_x)\beta_x \tilde{\mu} - 1 - \eta}{1 + \eta\sigma - \eta(\sigma - \gamma^{-1})(\beta_x + \chi(1 - \beta_x))} = 0.$$

This shows that an equilibrium with $\phi_x = 0$ arises when $\mu_a = \hat{\mu}_a$, where

$$\hat{\mu}_a \equiv \frac{1 + \eta}{\beta_x(1 + \eta\gamma^{-1}\beta_x)} - \rho\gamma \frac{1 + \eta}{\gamma + \eta}.$$

However, $\hat{\mu}_a$ is also consistent with an equilibrium with $\phi_x \neq 0$. Summing up, if $\mu_a = \mu_a^0$ there is no stationary linear equilibrium; if $\mu_a = \mu_a^a$ there are two stationary linear equilibria, one with $\phi_x \neq 0$ and one with $\phi_x = 0$; if $\mu_a \in R/\{\mu_a^0, \hat{\mu}_a\}$, there is a unique stationary linear equilibrium.

8.3 Proof of Proposition 2

If $\sigma_e^2 = 0$ then $\beta_s = \delta_s = 1$ and $\beta_x = \delta_x = \delta_{\bar{x}} = 0$. Substituting in (44)-(47) gives

$$\begin{aligned} \phi_x &= -\frac{1 + \eta}{1 + \eta\sigma - \eta(\sigma - \gamma^{-1})\chi}, \\ \phi_s &= \tilde{\mu} + \frac{\eta(\sigma - \gamma^{-1})(1 - \chi)}{1 + \eta\gamma^{-1}}\phi_x, \end{aligned}$$

and

$$\begin{aligned} \psi_x &= 0, \quad \psi_{\bar{x}} = -\gamma^{-1}\phi_x, \\ \psi_s &= \gamma^{-1}(\tilde{\mu} - \phi_s) = -\gamma^{-1} \frac{\eta(\sigma - \gamma^{-1})(1 - \chi)}{1 + \eta\gamma^{-1}}\phi_x, \end{aligned}$$

and ψ_0 can be determined from (48). Notice that ϕ_s is the only coefficient which depends on μ_a (through $\tilde{\mu}$). However, the equilibrium allocation only depends on the consumption levels c_{it} and on the relative prices $p_{it} - p_t$, which are independent of ϕ_s .

8.4 Proof of Proposition 3

For the following derivations recall that under imperfect information all the coefficients $\beta_s, \beta_x, \delta_s, \delta_x, \delta_{\bar{x}}$ are in $(0, 1)$ and $\chi \in [0, 1]$. Differentiating (44) with respect to μ_a (recalling that $\tilde{\mu} = \mu_a + \gamma\psi_a$ and ψ_a is constant), gives

$$\frac{\partial \phi_x}{\partial \mu_a} = \frac{\beta_x + \eta\gamma^{-1}(\delta_x\beta_x + \delta_{\bar{x}}(\beta_x + \chi(1 - \beta_x)))}{1 + \eta\sigma - \eta(\sigma - \gamma^{-1})(\beta_x + \chi(1 - \beta_x))} > 0,$$

where the denominator is positive since $\beta_x + \chi(1 - \beta_x) < 1$ and $\eta(\sigma - \gamma^{-1}) < \eta\sigma$. Differentiating (??) gives

$$\frac{\partial \phi_s}{\partial \mu_a} = \frac{\beta_s + \eta\gamma^{-1}\delta_s}{1 + \eta\gamma^{-1}} + \left[\frac{\eta\gamma^{-1}(\delta_x + \delta_{\bar{x}}(1 - \chi))\beta_s}{1 + \eta\gamma^{-1}} + \frac{\eta\beta_s(\sigma - \gamma^{-1})(1 - \chi)}{1 + \eta\gamma^{-1}} \frac{\partial \phi_x}{\partial \mu_a} \right].$$

Some lengthy algebra, in the supplementary material, shows that the term in square brackets is positive, so $\partial \phi_s / \partial \mu_a > 0$. Next, differentiating (47) gives

$$\frac{\partial \psi_s}{\partial \mu_a} = \gamma^{-1} \left(\delta_s - \frac{\partial \phi_s}{\partial \mu_a} \right).$$

To prove that this expression is negative notice that

$$\frac{\partial \phi_s}{\partial \mu_a} > \frac{\beta_s + \eta\gamma^{-1}\delta_s}{1 + \eta\gamma^{-1}} > \delta_s,$$

where the last inequality follows from $\beta_s > \delta_s$, which follows immediately from (29) and (30). To prove that $\partial \psi_\theta / \partial \mu_a > 0$ it is sufficient to use of the last result together with Lemma 2, which immediately implies that $\partial \psi_\theta / \partial \mu_a = -(\sigma_e^2 / \sigma_\theta^2) \partial \psi_s / \partial \mu_a$.

8.5 Proof of Proposition 4

The argument in the text shows that there is a μ_a that gives coefficients $\psi_s = 0$ and $\psi_\theta = (1 + \eta) / (\gamma + \eta)$, if one assumes that consumers form expectations based on a_{t-1}, s_t, x_{it} , and \bar{x}_{it} . It remains to check that this value of μ_a is not equal to μ_a^0 , so that $\phi_x \neq 0$ and observed prices reveal \bar{x}_{it} . The algebra is presented in the supplementary material.

8.6 Proof of Lemma 3

Let me consider the case $\gamma \neq 1$, the proof for the case $\gamma = 1$ follows similar steps and is presented in the supplementary material. First, I derive expressions for the conditional expectations $E[C_{it}^{1-\gamma} | a_{t-1}]$ and $E[N_{it}^{1+\eta} | a_{t-1}]$. Substituting for c_{it} in the first, using (2), I obtain

$$\mathbb{E} \left[C_{it}^{1-\gamma} | a_{t-1} \right] = e^{(1-\gamma)(\psi_0 + \psi_a a_{t-1}) + \frac{1}{2}(1-\gamma)^2 (\psi_\theta^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2 + \psi_x^2 \sigma_e^2 + \psi_{\bar{x}}^2 \sigma_v^2)}.$$

Using (20) to substitute for d_{it} , and the fact that $a_{it} = a_t + \epsilon_{it}$ and $p_{it} - p_t = \phi_x \epsilon_{it}$, I derive the equilibrium labor supply

$$N_{it} = \frac{D_{it} P_{it}^{-\sigma}}{A_{it}} = e^{\kappa_d + \psi_0 + \psi_a a_{t-1} + \psi_\theta \theta_t + \psi_s \epsilon_t - a_t - (1 + \sigma \phi_x - (\psi_{\bar{x}} + \sigma \phi_x) \chi) \epsilon_{it}}. \quad (50)$$

From this expression, I obtain

$$\mathbb{E} \left[N_{it}^{1+\eta} | a_{t-1} \right] = e^{(1+\eta)(\kappa_d + \psi_0 + (\psi_a - \rho) a_{t-1}) + \frac{1}{2}(1+\eta)^2 ((\psi_\theta - 1)^2 \sigma_\theta^2 + \psi_s^2 \sigma_\epsilon^2 + (1 + \sigma \phi_x - (\psi_{\bar{x}} + \sigma \phi_x) \chi)^2 \sigma_\epsilon^2)}.$$

Using the fact that $\psi_a = \rho(1 + \eta) / (\gamma + \eta)$ to group the terms in a_{t-1} , the instantaneous conditional expected utility takes the form

$$\mathbb{E} [U(C_{it}, N_{it}) | a_{t-1}] = \left[\frac{1}{1 - \gamma} e^{(1-\gamma)(k_1 + \psi_0)} - \frac{1}{1 + \eta} e^{(1+\eta)(k_2 + \psi_0)} \right] e^{(1-\gamma)\psi_a a_{t-1}}. \quad (51)$$

where

$$k_1 = \frac{1}{2} (1 - \gamma) (\psi_\theta^2 \sigma_\theta^2 + \psi_s^2 \sigma_\epsilon^2 + \psi_x^2 \sigma_\epsilon^2 + \psi_{\bar{x}}^2 \sigma_v^2), \quad (52)$$

$$k_2 = \kappa_d + \frac{1}{2} (1 + \eta) \left((\psi_\theta - 1)^2 \sigma_\theta^2 + \psi_s^2 \sigma_\epsilon^2 + (1 + \sigma \phi_x - (\psi_{\bar{x}} + \sigma \phi_x) \chi)^2 \sigma_\epsilon^2 \right). \quad (53)$$

The equilibrium equation (48) shows that, for each value of μ_a , there is a one-to-one correspondence between τ and ψ_0 , and ψ_0 is the only equilibrium coefficient affected by τ . Therefore, if τ is set optimally, ψ_0 must maximize the term in square brackets on the right-hand side of (51). Solving this problem shows that the term in square brackets must be equal to $\exp\{((1 - \gamma)(1 + \eta)) / (\gamma + \eta)(k_1 - k_2)\}$ and

$$\psi_0 = \frac{(1 - \gamma) k_1 - (1 + \eta) k_2}{\gamma + \eta}. \quad (54)$$

Then, I can take the unconditional expectation of (51) and sum across periods to obtain

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E} [U(C_{it}, N_{it})] = \frac{\gamma + \eta}{(1 + \eta)(1 - \gamma)} e^{\frac{(1-\gamma)(1+\eta)}{\gamma+\eta}(k_1 - k_2)} \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[e^{\frac{(1-\gamma)(1+\eta)}{\gamma+\eta} \rho a_{t-1}} \right].$$

Letting

$$w \equiv k_1 - k_2, \quad (55)$$

and

$$W_0 \equiv \frac{\gamma + \eta}{1 + \eta} \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[e^{\frac{(1-\gamma)(1+\eta)}{\gamma+\eta} \rho a_{t-1}} \right],$$

I then obtain the expression in the text. Combining (52), (53), and (55), shows that w can be expressed as follows

$$\begin{aligned}
w &= \frac{1}{2} \frac{(1-\gamma)(1+\eta)}{\gamma+\eta} \sigma_\theta^2 - \frac{1}{2} (\gamma+\eta) \left(\left(\psi_\theta - \frac{1+\eta}{\gamma+\eta} \right)^2 \sigma_\theta^2 + \psi_s^2 \sigma_e^2 \right) + \\
&+ \frac{1}{2} (1-\gamma) (\psi_x^2 \sigma_\epsilon^2 + \psi_{\bar{x}}^2 \sigma_v^2) - \frac{1}{2} (1+\eta) (1 + \sigma \phi_x - (\psi_{\bar{x}} + \sigma \phi_x) \chi)^2 \sigma_\epsilon^2, \\
&- \frac{1}{2} \left(\psi_x^2 \sigma_\epsilon^2 + (\psi_{\bar{x}} + \sigma \phi_x)^2 \sigma_{v|\epsilon}^2 \right) + \frac{1}{2} \sigma (\sigma - 1) \phi_x^2 \sigma_{\epsilon|v}^2. \tag{56}
\end{aligned}$$

This shows that w is a quadratic functions of the equilibrium coefficients ϕ and ψ . Moreover, Proposition 3 shows that ϕ and ψ are linear functions of μ_a . Therefore, (56) implicitly defines w as a quadratic function of μ_a . The discounted sum in W_0 is always finite if $\rho < 1$, because each term is bounded by $|\exp\{(1-\gamma)\psi_a \rho a_{-1} + (1/2)((1-\gamma)^2/(1-\rho^2))\psi_a^2 \sigma_\theta^2\}|$. If, instead, $\rho = 1$, to ensure that the sum is finite it is necessary to assume that $\beta \exp\{(1/2)(1-\gamma)^2 \psi_a^2 \sigma_\theta^2\} < 1$, which is equivalent to the inequality in footnote 6.

8.7 Derivation of equations (17) and (18)

I will first show that (17) corresponds to (56) in the proof of Lemma 3. For ease of exposition, the expression in the text omits the constant term $-(1/2)((1-\gamma)(1+\eta)/(\gamma+\eta))\sigma_\theta^2$. The first two terms in (17) can be derived from the two terms after the constant in (56), simply using the definitions of c_t and c_{it} . Equation (50), can be used to derive $\int_0^1 (n_{it} - n_t)^2 di$, and check that the third term in (17) equals the third term after the constant in (56). Finally, the last line of (56) corresponds to $-\kappa_d$, by (22), while equation (50) implies that $n_t = \kappa_d + c_t - a_t$. This shows that the last terms in (17) and (56) are equal. To derive (18) notice that, as just argued, the last line of (56) is equal to $-\kappa_d$. The derivations in Lemma 5 can then be used to obtain the expression in the text.

8.8 Proof of Lemma 4

I concentrate on the case $\gamma \neq 1$, the case $\gamma = 1$ is proved along similar lines. Given two monetary policies $\mu'_a = \mu_a$ and $\mu''_a = \mu_a + u$, let C'_{it}, N'_{it} and C''_{it}, N''_{it} denote the associated equilibrium allocations, and define the function

$$f(\delta, u) \equiv \left\{ \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[e^{\frac{(1-\gamma)(1+\eta)}{\gamma+\eta} \rho a_{t-1}} \right] \right\}^{-1} \left\{ \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[U \left(e^\delta C'_{it}, N'_{it} \right) \right] - \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[U \left(C''_{it}, N''_{it} \right) \right] \right\}.$$

Proceeding in as in the proof of Lemma 3, it is possible to show that

$$f(\delta, u) = \frac{1}{1-\gamma} e^{(1-\gamma)(\delta+k_1+\psi_0)} - \frac{1}{1+\eta} e^{(1+\eta)(k_2+\psi_0)} - \frac{\gamma+\eta}{(1+\eta)(1-\gamma)} e^{\frac{(1+\eta)(1-\gamma)}{\gamma+\eta} w(\mu_a+u)},$$

where k_1 and k_2 are defined in (52) and (53), for the coefficients $\{\phi, \psi\}$ associated to the policy μ_a , and the function $w(\cdot)$ is defined by (56). Let the function $\delta(u)$ be defined implicitly by $f(\delta(u), u) = 0$. It is immediate that $\delta(0) = 0$. Moreover,

$$\begin{aligned} \left. \frac{\partial f(\delta, u)}{\partial \delta} \right|_{\delta=u=0} &= e^{(1-\gamma)(k_1+\psi_0)}, \\ \left. \frac{\partial f(\delta, u)}{\partial u} \right|_{\delta=u=0} &= -e^{\frac{(1+\eta)(1-\gamma)}{\gamma+\eta}w(\mu_a)} w'(\mu_a), \end{aligned}$$

and (54) implies that

$$e^{(1-\gamma)(k_1+\psi_0)} = -e^{\frac{(1+\eta)(1-\gamma)}{\gamma+\eta}w(\mu_a)}.$$

It follows that $\delta'(u) = w'(\mu_a)$. Since $\Delta(\mu_a, \mu_a + u) = \exp\{\delta(u)\} - 1$, by definition, the result follows from differentiating this expression at $u = 0$.

8.9 Proof of Proposition 5

Let me begin by setting up and characterizing the planner's problem. Then, I will show that there is a monetary policy that reaches the constrained optimal allocation. Let a_- be a given scalar representing productivity in the previous period. Let θ be a normally distributed random variable with mean zero and variance σ_θ^2 and let s be a random variable given by $s = \theta + e$, where e is also a normal random variable with mean zero and variance σ_e^2 . Let x, \bar{x} and \tilde{x} be random variables given by $x = \theta + \epsilon$, $\bar{x} = \theta + v$, and $\tilde{x} = \bar{x} + \tilde{\epsilon}$, where ϵ, v and $\tilde{\epsilon}$ are independent random variables with zero mean and variances $\sigma_\epsilon^2, \sigma_v^2, \sigma_{\tilde{\epsilon}}^2$. The planner's problem is to choose functions $\tilde{C}(s, x, \bar{x}, \tilde{x})$, $C(s, x, \bar{x})$, and $N(s, x, \theta)$ that maximize

$$\mathbb{E}[U(C(s, x, \bar{x}), N(s, x, \theta))]$$

subject to

$$C(s, x, \bar{x}) = \left(\mathbb{E} \left[\left(\tilde{C}(s, x, \bar{x}, \tilde{x}) \right)^{\frac{\sigma-1}{\sigma}} \mid s, x, \bar{x} \right] \right)^{\frac{\sigma}{\sigma-1}} \quad \text{for all } s, x, \bar{x}, \quad (57)$$

$$e^{\rho a_- + \tilde{x}} N(s, \tilde{x}, \theta) = \mathbb{E} \left[\tilde{C}(s, x, \bar{x}, \tilde{x}) \mid s, \tilde{x}, \theta \right] \quad \text{for all } s, \tilde{x}, \theta. \quad (58)$$

Let $\Lambda(s, \tilde{x}, \theta)$ denote the Lagrange multiplier on constraint (58). Substituting (57) in the objective function, one obtains the following first-order conditions with respect to $\tilde{C}(s, x, \bar{x}, \tilde{x})$ and $N(s, \tilde{x}, \theta)$:

$$\left(\tilde{C}(s, x, \bar{x}, \tilde{x}) \right)^{-\frac{1}{\sigma}} (C(s, x, \bar{x}))^{\frac{1}{\sigma}-\gamma} = \mathbb{E}[\Lambda(s, \tilde{x}, \theta) \mid s, x, \bar{x}, \tilde{x}], \quad (59)$$

$$(N(s, \tilde{x}, \theta))^\eta = e^{\rho a_- + \tilde{x}} \Lambda(s, \tilde{x}, \theta). \quad (60)$$

The planner's problem is concave, so (59) and (60) are both necessary and sufficient for an optimum. To prove the proposition, I take the equilibrium allocation associated to a generic pair (μ_a, τ) , and I derive conditions on μ_a and τ which ensure that it satisfies (59) and (60). An equilibrium allocation immediately satisfies the constraints (57) and (58), the first by construction, the second by market clearing. Take a linear equilibrium allocation characterized by φ and ψ . Let $\tilde{C}(\cdot, \cdot, \cdot, \cdot)$ and $N(\cdot, \cdot, \cdot)$ take the form

$$\tilde{C}(s, x, \bar{x}, \tilde{x}) = \exp \{ \sigma \kappa_{\bar{p}} + \psi_0 + \psi_a a_- + \psi_s s + \psi_x x + \psi_{\bar{x}} \bar{x} - \sigma \phi_x (\tilde{x} - \bar{x}) \}, \quad (61)$$

$$N(s, \tilde{x}, \theta) = \exp \{ \kappa_d + \psi_0 + \psi_a a_- + \psi_s s + (\psi_x + \psi_{\bar{x}}) \theta - \rho a_- - \tilde{x} - (\sigma \phi_x - (\psi_{\bar{x}} + \sigma \phi_x) \chi) (\tilde{x} - \theta) \}$$

I conjecture that the Lagrange multiplier $\Lambda(s, \tilde{x}, \theta)$ takes the log-linear form

$$\Lambda(s, \tilde{x}, \theta) = \exp \{ \lambda_0 + \lambda_s s + \lambda_x \tilde{x} + \lambda_\theta \theta \}. \quad (62)$$

Let me first check the first-order condition for consumption, (59). Substituting (61) in (57) and using the definition of $\kappa_{\bar{p}}$, I get

$$C(s, x, \bar{x}) = \exp \{ \psi_0 + \psi_a a_- + \psi_s s + \psi_x x + \psi_{\bar{x}} \bar{x} \}.$$

After some simplifications, the left-hand side of (59) becomes

$$\tilde{C}(s, x, \bar{x}, \tilde{x})^{-\frac{1}{\sigma}} C(s, x, \bar{x})^{\frac{1}{\sigma} - \gamma} = \exp \{ -\kappa_{\bar{p}} + \phi_x (\tilde{x} - \bar{x}) \} \exp \{ -\gamma (\psi_0 + \psi_a a_- + \psi_s s + \psi_x x + \psi_{\bar{x}} \bar{x}) \}.$$

The right-hand side of (59), using (62), is equal to

$$\mathbb{E} [\Lambda(s, \tilde{x}, \theta) | s, x, \bar{x}, \tilde{x}] = \exp \left\{ \lambda_0 + \lambda_s s + \lambda_x \tilde{x} + \lambda_\theta \mathbb{E} [\theta | s, x, \bar{x}] + \frac{1}{2} \lambda_\theta^2 \hat{\sigma}_\theta^2 \right\},$$

where $\hat{\sigma}_\theta^2$ is the residual variance of θ , equal to $(\pi_\theta + \pi_s + \pi_x + \pi_{\bar{x}})^{-1}$. Therefore, to ensure that (59) holds for all s, x, \bar{x}, \tilde{x} , the following conditions must hold,

$$\begin{aligned} \lambda_0 + \frac{1}{2} \lambda_\theta^2 \hat{\sigma}_\theta^2 &= -\kappa_{\bar{p}} - \gamma (\psi_0 + \psi_a a_-), \\ \lambda_s + \lambda_\theta \delta_s &= -\gamma \psi_s, \\ \lambda_x &= \phi_x, \\ \lambda_\theta \delta_x &= -\gamma \psi_x, \\ \lambda_\theta \delta_{\bar{x}} &= -\gamma \psi_{\bar{x}} - \phi_x. \end{aligned}$$

Set $\lambda_\theta = -(\phi_a + \gamma \psi_a)$, $\lambda_s = \phi_s$, $\lambda_x = \phi_x$ and $\lambda_0 = -\kappa_{\bar{p}} - \gamma (\psi_0 + \psi_a a_-) - (1/2) \lambda_\theta^2 \hat{\sigma}_\theta^2$. Then, the first and the third of these conditions hold immediately. The other three follow from the

equilibrium relations (36)-(38). Let me now check the first order condition for labor effort, (60). Substituting (62) and matching the coefficients on both sides, gives

$$\begin{aligned}\lambda_0 + \rho a_- &= \eta (\kappa_d + \psi_0 + (\psi_a - \rho) a_-), \\ \lambda_s &= \eta \psi_s, \\ \lambda_x + 1 &= -\eta (1 + \sigma \phi_x - (\psi_{\bar{x}} + \sigma \phi_x) \chi), \\ \lambda_\theta &= \eta (\psi_x + \psi_{\bar{x}} + \sigma \phi_x - (\psi_{\bar{x}} + \sigma \phi_x) \chi).\end{aligned}$$

Substituting, the λ 's derived above, using $\psi_a = \rho(1 + \eta) / (\gamma + \eta)$ and rearranging, gives

$$(\gamma + \eta) \psi_0 + \eta \kappa_d + \kappa_{\bar{p}} + (1/2) (\phi_a + \gamma \psi_a)^2 \hat{\sigma}_\theta^2 = 0, \quad (63)$$

$$\phi_s - \eta \psi_s = 0, \quad (64)$$

$$(1 + \eta \sigma) \phi_x + 1 + \eta - \eta (\psi_{\bar{x}} + \sigma \phi_x) \chi = 0, \quad (65)$$

$$\phi_a + \gamma \psi_a + \eta (\psi_x + \psi_{\bar{x}} + \sigma \phi_x - (\psi_{\bar{x}} + \sigma \phi_x) \chi) = 0. \quad (66)$$

To complete the proof, I need to find μ_a and τ such that the corresponding equilibrium coefficients φ and ψ satisfy (63)-(66). With the notation $\tilde{\mu} = \mu_a + \gamma \psi_a$, setting $\tilde{\mu}$ equal to

$$\tilde{\mu}^* \equiv \frac{\eta (\sigma - \gamma^{-1}) (1 - \chi) (1 + \eta)}{(1 + \eta \gamma^{-1} (\delta_x + \delta_{\bar{x}} (1 - \chi))) (1 + \eta \sigma - \eta (\sigma - \gamma^{-1}) \chi) + \eta^2 (\sigma - \gamma^{-1}) (1 - \chi) \gamma^{-1} \delta_{\bar{x}} \chi}, \quad (67)$$

ensures that (64)-(66) are satisfied. To see why these three conditions can be jointly satisfied, notice that the equilibrium conditions (34) and (35) can be rewritten as

$$\begin{aligned}\phi_s - \eta \psi_s &= [\eta ((\psi_x + \psi_{\bar{x}} + \sigma \phi_x) - (\psi_{\bar{x}} + \sigma \phi_x) \chi) + \phi_a + \gamma \psi_a] \beta_s, \\ (1 + \eta \sigma) \phi_x + 1 + \eta - \eta (\psi_{\bar{x}} + \sigma \phi_x) \chi &= [\eta ((\psi_x + \psi_{\bar{x}} + \sigma \phi_x) - (\psi_{\bar{x}} + \sigma \phi_x) \chi) + \phi_a + \gamma \psi_a] \beta_x,\end{aligned}$$

so that, in equilibrium, (66) implies the other two. Finally, the subsidy τ can be set so as to ensure that (63) is satisfied. The value of ϕ_x at the optimal monetary policy is

$$\phi_x^* = \frac{-1 - \eta + \eta \gamma^{-1} \delta_{\bar{x}} \chi \tilde{\mu}^*}{1 + \eta \sigma - \eta (\sigma - \gamma^{-1}) \chi}. \quad (68)$$

Substituting (67) in the expression $\eta \gamma^{-1} \delta_{\bar{x}} \chi \tilde{\mu}^*$, shows that this expression is strictly smaller than $1 + \eta$, which implies that $\phi_x^* < 0$. This shows that $\mu_a^* \neq \mu_a^0$, so that, by Proposition 1, the associated coefficients φ^* and ψ^* form a linear equilibrium.

8.10 Proof of Proposition 6

Let me derive the value of ψ_s at the constrained efficient allocation. From condition (64) and the equilibrium condition $\phi_s + \gamma\psi_s = \tilde{\mu}\delta_s$, I get

$$\psi_s^* = \frac{1}{\gamma + \eta} \delta_s \tilde{\mu}^*.$$

If $\chi = 0$, the consumer extracts perfect information from $\bar{x}_{it} = \theta_t$ and $\delta_s = 0$, which implies that $\psi_s^* = 0$. If, instead $\chi > 0$, ψ_s^* inherits the sign of $\tilde{\mu}^*$. Inspecting (67) shows that if $\eta > 0$, $\chi < 1$ and $\sigma\gamma \neq 1$, $\tilde{\mu}^*$ is not zero and has the sign of $\sigma\gamma - 1$. In all other cases, $\tilde{\mu}^* = 0$. Therefore, if $\eta > 0$, $\chi \in (0, 1)$ and $\sigma\gamma \neq 1$, ψ_s^* is not zero and has the sign of $\sigma\gamma - 1$. In all remaining cases $\psi_s^* = 0$. The inequalities for ψ_θ^* follow from Lemma 2. To prove the inequalities for μ_a^* , notice that, by Proposition 3 there is a decreasing relation between μ_a and ψ_s , and $\psi_s = 0$ at $\mu_a = \mu_a^{fs}$.

8.11 Proof of Proposition 7

The first part of the Proposition is proved by the two examples discussed in the text. Let me prove the second part. By Proposition 5, social welfare under the optimal monetary policy is the value of a single decision maker's optimization problem (the planner's). For a single decision maker, increasing the variance σ_e^2 is equivalent to observing the signal $s_t + \zeta_t$ instead of s_t , where ζ_t is an additional independent error with variance σ_ζ^2 equal to the increase in σ_e^2 . That is, a decision maker who observes s_t can always replicate the payoff of a decision maker with a less precise signal, by adding random noise to s_t and following the associated optimal policy. Therefore, the decision maker's payoff cannot increase when σ_e^2 increases.

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